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On the Robustness of Multidimensional Counting Poverty Orderings

José Gallegos
University of Piura

Gaston Yalonetzky
University of Leeds

Francisco Azpitarte
Melbourne Institute, University of Melbourne & Brotherhood of St Laurence

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NON-TECHNICAL SUMMARY

Multidimensional poverty measures based on counts of dimensions in which individuals are deprived have gained prominence in recent decades. Poverty measures of this sort are currently used by many governments and international organisations to monitor poverty trends in developed and developing countries.

Defining poverty in this way is a very simple and intuitive approach. Yet when constructing counting poverty measures analysts face multiple methodological choices that can influence poverty levels and comparisons. These choices include the function linking individuals' level of deprivation with the number of poverty dimensions, the threshold specifying the minimum number of dimensions individuals need to be deprived to be deemed as multidimensionally poor, and the weights assigned to each of the wellbeing indicators.

While the sensitivity of poverty estimates to these choices is generally acknowledged, the common approach involves evaluating the sensitivity of poverty orderings considering a limited and usually arbitrarily set of alternative individual poverty functions, cut-offs values and dimensional weights. Although easy to implement, this approach is inferior to classical approaches used in the income poverty literature.

This paper proposes new dominance criteria for multidimensional counting poverty measures. We derived conditions that are both necessary and sufficient to guarantee the robustness of multidimensional poverty orderings to the choice of the poverty index, the multidimensional poverty cut-off, and the vector of dimensional weights used to construct counting poverty scores. The new conditions are easy to test empirically, and the new criteria apply to a broad class of contemporary counting poverty measures.

We also derived a set of useful necessary conditions that allow the analyst to rule out the robustness of poverty comparisons to changes in poverty functions, identification cut-offs, and dimensional weights. These conditions are easy to implement, as they only require comparing the proportion of people deprived in each of the dimensions and the proportion deprived in all dimensions.

We illustrate our method through an empirical assessment of poverty trends in Australia in the 2000s using a framework based on three indicators of economic deprivation. Our findings indicate that poverty comparisons based on counting measures can be highly sensitive to changes in dimensional weights, cut-offs and poverty functions. Given the growing prominence of this type of measures in social policy and academic debates, it is crucial to have dominance conditions that allow the systematic evaluation of poverty orderings to changes in those methodological choices. This papers constitutes as important step in this direction.

ABOUT THE AUTHORS

José Gallegos is the director of the Impact Evaluation Office at the Ministry of Production in Peru. He holds a PhD in Economics from Syracuse University. Currently he is also Research Associate at the Research Centre of Pacifico University (Lima) and part-time lecturer at Piura University (Lima campus). Besides impact evaluation and econometric methods, his interests also include household economics and poverty measurement. Email: Jvgallegos@gmail.com

Gaston Yalonetzky works as Lecturer at the Leeds University Business School, University of Leeds. Before holding that position he worked as Research Officer at Oxford Poverty and Human Development Initiative, University of Oxford. He holds a Masters degree in development economics and a PhD in Economics from the University of Oxford. His research interests include distributional analysis, public and welfare economics and economic development. Email: G.Yalonetzky@leeds.ac.uk

Francisco Azpitarte is the Ronald Henderson Research Fellow at Melbourne Institute, University of Melbourne and the Brotherhood of St Laurence. He holds a Masters degree in Economic Analysis from the Universitat Autonoma de Barcelona and completed his PhD in Economics at the Universidade de Vigo in 2009. His research interests are the analysis of poverty and inequality and the impact of poverty on human development. Email: fraz@unimelb.edu.au

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(ARC Centre of Excellence for Children and Families over the Life Course)
Institute for Social Science Research, The University of Queensland (administration node)
UQ Long Pocket Precinct, Indooroopilly, Qld 4068, Telephone: +61 7 334 67477
Email: lcc@uq.edu.au, Web: www.lifecoursecentre.org.au

Abstract

Counting poverty measures have gained prominence in the analysis of multidimensional poverty in recent decades. However, poverty orderings based on these measures typically depend on methodological choices regarding poverty indices, poverty cut-offs, and dimensional weights whose impact on poverty rankings is often not well understood. In this paper we propose new dominance conditions that allow the analyst to evaluate the robustness of poverty comparisons to those choices. These conditions provide an approach to the evaluation of the sensitivity of poverty orderings superior to the common approach of considering a restricted and arbitrary set of indices, cut-offs, and weights. The new criteria apply to a broad class of counting poverty measures widely used in empirical analysis including the class of measures proposed by Alkire and Foster (2011), the class proposed by Chakravarty and D'Ambrosio (2006), and combinations thereof. We illustrate these methods with an application to multidimensional poverty in Australia in the 2000s.

Keywords: multidimensional poverty; counting measures; dominance conditions; Australia

1 Introduction

Multidimensional counting poverty measures are widely used by academics and policymakers around the world. Since the works of Atkinson (2003), Chakravarty and D'Ambrosio (2006), and Alkire and Foster (2011), many governments and international institutions have adopted counting poverty measures in order to monitor poverty trends in developed and developing countries alike. Recent examples include World Bank (2016), the "Multidimensional Poverty Index" used by UNDP in its Human Development Reports since UNDP (2010), and the measures of people at risk of poverty and social exclusion currently used by Eurostat to assess living conditions in Europe (Eurostat, 2014). Moreover, the governments of Bhutan, Brazil, China, Colombia, El Salvador, Honduras, Malaysia, and Mexico, have already incorporated this type of measures into their set of national statistics.[1] Meanwhile, other countries are expressing interest toward future adoption.[2]

Poverty evaluations based on those measures depend on a range of arbitrary choices that are likely to influence poverty comparisons. These choices include the specific properties of the poverty function, the rule employed to identify the multidimensionally poor, and the weights assigned to each of the different dimensions or indicators. While the sensitivity of poverty estimates to these choices is generally acknowledged, the common approach in the literature proceeds by evaluating the sensitivity of poverty orderings considering a limited and usually arbitrarily chosen set of alternative indices, weights, and cut-offs (e.g., see Nusbaumer et al., 2012; Alkire and Santos, 2014). Although easy to implement, this type of approach is inferior to the stochastic dominance approach commonly used in the income poverty literature, which reduces the problem of testing the robustness of alternative choices over a large, usually continuous domain, to a smaller set of finite distributional comparisons. Notwithstanding their widespread consideration in distributional analysis, including monetary poverty research, the use of dominance conditions for evaluating the robustness of counting poverty orderings to alternative methodological choices is still rare. However, the soaring popularity of counting poverty measures, together with their reliance on a range of arbitrary methodological choices, justifies the development of testable conditions for gauging the robustness of poverty comparisons based on the counting approach.

Existing proposals have succeeded in providing robustness tests based on changing a handful of key sets of parametric or functional choices while keeping the others constant. For example, in the case of counting poverty measures, Lasso de la Vega (2010) showed how to test for the robustness of comparisons to alterna-

[1]See:$www.ophi.org.uk/policy/national-policy/$.

[2]See:$www.ophi.org.uk/government-of-spain-calls-for-the-adoption-of-a-multidimensional-poverty-index-post-2015/$.

tive poverty identification cut-offs and individual poverty functions, while keeping several other parameters constant (poverty lines and dimensional weights). More recently, Yalonetzky (2014) proposed a robustness test for ordinal variables, alternative functional forms, deprivation lines and weights, but only useful for extreme poverty identification approaches (union and intersection). Likewise, Permanyer and Hussain (2017) proposed a highly flexible robustness test based on first-order dominance conditions applied to multiple binary variables, but working under a union approach to poverty identification. In this paper, we propose complementary dominance conditions whose fulfillment guarantees the robustness of comparisons to broad alternative combinations of functional forms (for individual poverty measures), deprivation weights *and* counting poverty identification criteria.

A similar concern prevails among users of composite indices regarding the robustness of comparisons to alternative choices of weights used to aggregate the wellbeing indicators. The recent contributions of Permanyer (2011) and Foster et al. (2013) provide innovative methods to gauge the degree of robustness of both pairwise comparisons and country rankings to alternative choices of weights. While these methods are well suited to study comparisons using multidimensional measures of welfare, they have not yet been adapted to the context of multidimensional poverty measures in which additional complicating measurement choices play key roles, e.g. deprivation lines and multidimensional poverty cut-offs which help identify the poor, and poverty intensity functions. Moreover, these methods provide measures of comparisons' degree of robustness to a subset of weights defined around a pre-specified vector of weights (e.g. equal weights). Finally, while these methods are useful to explore robustness across a subset of weights, they do not solve the key computational problem addressed by stochastic dominance techniques mentioned above; namely, how to transform a robustness test over a large continuous domain into a smaller set of finite distributional comparisons. For these stated reasons, we do not pursue these robustness methods in this paper, instead favouring a stochastic dominance approach.

Counting poverty measures focusing on the number of dimensions in which individuals experience deprivation have a long tradition in the poverty literature.[3] These measures share key features with measures based on the social welfare and axiomatic approaches to multidimensional deprivation that can be discussed in a common framework. Indeed, as shown by Atkinson (2003), dominance conditions in these approaches necessarily involve the comparison of the groups deprived in *any* and *all* dimensions. In a very influential paper, Alkire and Foster (2011)

[3]As cited in Atkinson (2003), early applications of counting measures include the works by Townsend (1979) for the United Kingdom, Erikson (1993) for Sweden, and Callan et al. (1999) for Ireland. More recent applications and methodological innovations include Chakravarty and D'Ambrosio (2006), Bossert et al. (2009), Alkire and Foster (2011), and Permanyer (2014).

proposed a new method that combines the counting and axiomatic approaches to the measurement of multidimensional deprivation. In this approach, the poor are identified using a weighted counting measure and a poverty cut-off representing the minimum value of this weighted counting measure required to be classified as poor.[4] The deprivations of the poor are then aggregated using a measure of the Foster–Greer–Thobercke family of poverty measures (Foster et al., 1984). The resulting poverty measures satisfy standard axioms of multidimensional poverty measurement often invoked in the literature.[5]

This paper contributes to the existing literature by proposing new dominance criteria for multidimensional counting poverty measures. We derive conditions that are both necessary and sufficient to guarantee the robustness of multidimensional poverty orderings to the choice of the poverty index, the multidimensional poverty cut-off, and the vector of dimensional weights used to construct counting poverty scores. The new conditions are easy to test empirically as they involve the comparison of frequencies of people deprived in different sets of dimensions. For example, comparing the proportion of people deprived only in electricity in country A against their equally-deprived counterparts in country B, comparing the proportion deprived only in electricity and sanitation in A versus B, and so forth. Importantly, the new criteria apply to a broad class of counting poverty measures including the classes of measures proposed by Chakravarty and D'Ambrosio (2006), Alkire and Foster (2011), and combinations thereof; in turn including the multidimensional headcount and the adjusted headcount ratio indices widely used in poverty research.

Our results build on the conditions proposed by Lasso de la Vega (2010) to identify unambiguous rankings for a class of poverty indices and poverty cut-offs. Our results extend hers in a number of ways. Firstly, while conditions in Lasso de la Vega (2010) apply *only to a particular vector of deprivation weights*, our new conditions guarantee the robustness of counting poverty orderings to changes in poverty indices and cut-offs *for any conceivable vector of dimensional weights*. Furthermore we derive a set of useful necessary conditions that allow the analyst to rule out the robustness of poverty comparisons to changes in poverty functions, identification cut-offs, and dimensional weights. These conditions require only the comparison of the proportion of people deprived in each of the dimensions and the proportion of people deprived in all dimensions. We propose statistical tests for the new dominance conditions based on the testing framework for pair-wise population comparisons proposed by Dardanoni and Forcina (1999) and Hasler (2007).

[4]When these weights are equal, the poverty cut-off can be interpreted as the minimum number of deprived dimensions required to be classified as poor.

[5]Key references in this literature also include Tsui (2002), Bourguignon and Chakravarty (2003), Duclos et al. (2006), and Permanyer (2014).

On the whole, we argue that the analytical methods proposed in this paper contribute significantly to the existing toolkit of robustness evaluation techniques for counting poverty orderings by covering combinations of parametric choices hitherto unavailable in the literature. We further discuss the scope and limitations of our proposal in the following sections.

We illustrate the new dominance conditions with an empirical assessment of poverty trends in Australia during the years 2002, 2006, 2010, a period which first saw improved monetary living standards in association with the commodity boom, followed by some decline in the aftermath of the financial crisis. Multidimensional poverty in Australia declined between 2002 and 2006, assuming equal weights. This was followed by an increase in poverty from 2006 to 2010 although poverty levels by 2010 remained below those in 2002. These results are robust to alternative poverty indices and poverty cut-offs. However, the new robustness conditions enable us to conclude that the 2002-2010 and 2006-2010 comparisons are not robust to changes in weights as the ordering of those years depends on the particular choice of dimensional weights and poverty cut-offs. By contrast, the reduction in multidimensional poverty between 2002 and 2006 was fully robust not only to a very wide range of choices of poverty index and poverty identification cut-offs, but also to any choice of dimensional weights.

The rest of the paper proceeds as follows. The next section presents the measurement framework and the class of counting poverty measures considered in the analysis. Key poverty statistics and some notation relevant for the derivation of the dominance results are also discussed in this section. The third section discusses the existing dominance conditions and develops the new dominance results for counting poverty measures. The fourth section briefly explains the statistical tests. The fifth section provides the empirical illustrations on multidimensional poverty reduction in Australia. Finally, the paper concludes with some remarks.

2 The Counting Approach to Poverty Measurement

2.1 Measurement Framework

We consider a population with N individuals and $D > 1$ indicators of wellbeing. Let X be a matrix of attainments where the typical element x_{nd} denotes the level of attainment by individual n on dimension d. If $x_{nd} < z_d$, where z_d is a deprivation line for dimension d from a D-dimensional vector of deprivation lines, Z, then we say that individual n is deprived in indicator d. Let $y_{nd} = \mathbb{I}(x_{nd} < z_d)$ where $\mathbb{I}$ is the indicator function that takes value 1 if the argument in parenthesis is true, and 0 otherwise. Therefore the matrix Y with dimensions $N \times D$ and typical element y_{nd}

translates the attainments into an identification of deprivations across dimensions and individuals. Here we must emphasize that there are different ways of defining the elements of matrix Y ranging from simple binary comparisons to complex logical operations. For example, y_{nd} could be a binary indicator of access to electricity where $y_{nd} = 1$ could denote access and $y_{nd} = 0$ would mean lack of access. On the other extreme, y_{nd} could also be a complex binary indicator taking the value of 1 whenever a set of conditions are fulfilled to a partial or full extent. For example, we could say that $y_{nd} = 1$ if at least one construction material (e.g. for floor, walls, roof, etc.) is of substandard quality, otherwise $y_{nd} = 0$ (i.e. a type of union approach for y_{nd}). But we could also say that $y_{nd} = 1$ if every adult in the family is illiterate, otherwise $y_{nd} = 0$ (i.e. a type of intersection approach for $y_{nd} = 0$). Unlike the proposals by Yalonetzky (2014) and Permanyer and Hussain (2017), the dominance conditions proposed in this paper do not apply directly to the joint distribution of the variables whose logical combinations lead to matrix Y. Our conditions build from Y once the rules used to construct the matrix of deprivations are set. Therefore a change in those rules would require implementing our proposed tests (or any other tests taking the construction of Y for granted) again. [6]

In order to account for the breadth of deprivations, most counting measures rely on individual deprivation scores defined as a weighted count of deprivations. Let $W := (w_1, w_2, ..., w_D)$ denote the vector of dimensional weights such that $w_d \geq 0 \ \wedge \ \sum_{d=1}^{D} w_d = 1$. The deprivation score for individual n is given by

$$c_n \equiv \sum_{d=1}^{D} w_d y_{nd},$$

There is only one vector of possible values of c_n for each particular choice of deprivation lines and weights. Moreover it is easy to show that the maximum number of possible values is given by: $\sum_{i=0}^{D} \binom{D}{i} = 2^D$. The vector of possible values is defined as: $V := (v_1, v_2, ..., v_l)$, where $\max l = 2^D$, $v_i < v_{i+j}$, $v_1 = 0$ and $v_l = 1$. [7]

[6] Another potential complication in the construction of matrices X and Y is that some attainments or deprivations may not always be observable, either directly or indirectly. This will often depend on the choice and definition of well-being indicator, as well as the degree of complexity of the decision rules used to define deprivations based on several indicators. For example, if a cohabiting couple is surveyed too early into their partnership before they have children, one may be unable to report the health issues affecting the children. Likewise, if one or two household heads are surveyed too late into their lives, one may be unable to retrieve information about the education of children in the household if their children do not live with them anymore already, unless the heads are asked explicitly about their offspring in retrospect. This is a challenge common to the literature, e.g. it would affect indices like the UNDP's MPI (see Alkire and Santos, 2014, table 1). We thank a referee for pointing out this issue.

[7] As shown by Permanyer (2014, table 1), alternative forms for the deprivation score are possible when variables are cardinal and not partitioned or dichotomised. However this diversity significantly contracts when we work with binary deprivation indicators, as is the case in the counting framework.

Following Alkire and Foster (2011) we characterise the set of multidimensionally poor with an identification rule $\rho_k(c_n)$ that equals 1 when the individual is poor and 0 otherwise. The indicator function ρ_k compares individuals' c_n with a multidimensional cut-off $k \in [0,1] \subset \mathbb{R}_+$ so that any person n is deemed to be poor if and only if $c_n \geq k$. As shown in Lasso de la Vega (2010), the function ρ_k is the only identification rule that satisfies the property of poverty consistency which requires $\rho_k(c_{n'}) = 1$ whenever $\rho_k(c_n) = 1$ and $c_n \leq c_{n'}$.[8]

Let $P(Y;W,k)$ denote a social poverty counting measure depending on the vector of deprivations, Y, the vector of weights used to construct the scores W, and the identification rule, and the cut-off k used for the identification rule $\rho_k(c_n)$. Even though the extent of individual deprivations also depends on the vector of dimensional poverty lines, Z, we do not include the latter in $P()$ for the sake of notational simplicity. Following Lasso de la Vega (2010), we consider a broad family of social poverty measures satisfying standard axioms in the literature on poverty measurement including:

Axiom 1. *Focus (FOC): $P(Y;W,k)$ should not be affected by changes in the deprivation score of a non-poor person as long as for this person it is always the case that: $c_n < k$.*

Axiom 2. *Monotonicity (MON): $P(Y;W,k)$ should increase whenever c_n increases and n is poor.*

Axiom 3. *Symmetry (SYM): $P(Y;W,k)$ should not be affected by permutations in the vector C of poverty scores c_n, i.e., $P(C,\rho_k) = P(C',\rho_k)$ where C' is any permutation of C.*

Axiom 4. *Population-replication invariance (PRI): $P(Y;W,k) = P(Y_R;W,k)$ where $Y_R = (Y,Y,...,Y)$ is any replication of the N rows of the deprivation matrix Y.*

Axiom 5. *Distribution sensitivity (DS): Let $c_j > c_i$ and let Y' be the vector of deprivations obtained from Y by removing a subset of deprivations from individual j in Y, and Y'' be the vector obtained from Y by removing the* same *subset of deprivations from individual i in Y. Then: $P(Y;W,k) - P(Y';W,k) > P(Y;W,k) - P(Y'';W,k)$.*

Note that axiom DS essentially prioritises the reduction of deprivation scores among those with higher initial deprivation scores, i.e. the poorest among the

[8] Recently, Permanyer and Riffe (2015) have proposed a broad class of identification rules, which mostly do not require comparing a weighted count of deprivations against a cut-off (rather these rules are based on a host of different logical operations). In fact, as the authors show, the counting identification rule introduced by Alkire and Foster (2011) and axiomatically characterised by Lasso de la Vega (2010) is a special member of the broader class of poverty identification functions. The dominance conditions proposed in this paper are specifically tailored for identification rules consistent with the dominance conditions derived by Lasso de la Vega (2010).

poor. We denote by $\mathbb{P}_1$ the class of social poverty counting measures P satisfying FOC, MON, SYM, and PRI. And let $\mathbb{P}_2 \subset \mathbb{P}_1$ denote the class of social poverty measures satisfying DS in addition to those four axioms. In this paper we propose dominance conditions for these two classes of poverty measures.

The following poverty statistics are important for the derivation of the dominance conditions. The multidimensional poverty headcount is widely used in poverty analysis based on counting measures and is given by:

$$H(Y;W,k) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k). \tag{1}$$

The measure $H(Y;W,k)$ provides the proportion of people whose poverty score c_n is at least as high as the multidimensional poverty cut-off k. This is a crude measure of poverty that fails to satisfy the monotonicity axiom as it does not take into account the depth of poverty. However, as shown in Lasso de la Vega (2010), even if $H(Y;W,k)$ does not belong to the classes $\mathbb{P}_1$ and $\mathbb{P}_2$ of poverty measures, the orderings based on the $H(Y;W,k)$ statistic for all k are useful to identify unambiguous rankings within the class $\mathbb{P}_1$.

We also use the adjusted headcount ratio proposed by Alkire and Foster (2011) which can be expressed as:

$$M(Y;W,k) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k)c_n. \tag{2}$$

The statistic $M(Y;W,k)$ is defined as the censored population average score, in which the censorship trait stems from setting the scores of non-poor people to zero, in order to fulfil the focus axiom. It is also known in the literature as the *adjusted headcount ratio* (Alkire and Foster, 2011). In contrast with $H(Y;W,k)$, $M(Y;W,k)$ takes into account the breadth of deprivation to characterise the overall level of poverty. The measure $M(Y;W,k)$ fails to satisfy the Distribution Sensitivity axiom and therefore does not belong in the class $\mathbb{P}_2$. However, as we discuss below, unambiguous orderings with respect to $M(Y;W,k)$ for all k imply robust orderings within the class $\mathbb{P}_2$.

To derive the new dominance conditions it is also useful to consider the *uncensored deprivation headcount*, which measures the proportion of people deprived in dimension d irrespective of their deprivation in other dimensions:

$$U_d(Y) \equiv \frac{1}{N}\sum_{n=1}^{N} y_{nd}. \tag{3}$$

3 Dominance Conditions for Counting Measures

In this section we present the new dominance conditions to assess the robustness of counting poverty orderings within the classes of poverty measures $\mathbb{P}_1$ and $\mathbb{P}_2$. These conditions build on the dominance results derived by Lasso de la Vega (2010). Let $P(A;W,k)$ and $P(B;W,k)$ refer to the social poverty indices of populations A and B, respectively, and let $H(A;W,k)$ and $H(B;W,k)$ refer to their multidimensional headcounts. The following result sets out the conditions for unambiguous poverty orderings within the class $\mathbb{P}_1$:

Condition 1. *$P(A;W,k) < P(B;W,k)$ for all P in $\mathbb{P}_1$ and any identification cutoff, k, if and only if $H(A;W,k) \leq H(B;W,k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | H(A;W,k) < H(B;W,k)$.*

Proof. See Lasso de la Vega (2010). □

Condition (1) states that poverty comparisons of A and B are robust to the choice of the poverty function satisfying FOC, MON, SYM, and PRI only when the ordering of headcount measures is the same *for every relevant value of* k.

Now let $M(A;W,k)$ and $M(B;W,k)$ refer to the adjusted headcount ratio of populations A and B, respectively. The following result establishes the conditions for unambiguous poverty rankings within the class $\mathbb{P}_2$:

Condition 2. *$P(A;W,k) < P(B;W,k)$ for all P in $\mathbb{P}_2$ and any identification cutoff, k, if and only if $M(A;W,k) \leq M(B;W,k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | M(A;W,k) < M(B;W,k)$.*

Proof. See Lasso de la Vega (2010) and Chakravarty and Zoli (2009). □

Thus, when the adjusted headcount ratio in population A is lower than in B *for every relevant value of* k then we can claim that poverty in A is lower than in B for any inequality-sensitive poverty measure in $\mathbb{P}_2$ satisfying DS. The following remark links condition (1) to (2):

Remark 1. *If $H(A;W,k) \leq H(B;W,k) \quad \forall k \in [0,1] \quad \wedge \exists k | H(A;W,k) < H(B;W,k)$ then $M(A;W,k) \leq M(B;W,k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | M(A;W,k) < M(B;W,k)$.*

Proof. See Alkire and Foster (2011, Theorem 2). □

Remark (1) states that the existence of dominance within the class $\mathbb{P}_1$ implies dominance within the class $\mathbb{P}_2$, which is not surprising given that $\mathbb{P}_2 \subset \mathbb{P}_1$. Conditions (1) and (2) can also be restricted to apply only to a subset of relevant k values, ruling out the lowest ones below a minimum k_{min}. In order to proceed this way, we construct censored deprivation scores such that: $c_n = 0$ whenever $c_n < k_{min}$. Then conditions (1) and (2) apply only to those P which rule out poverty identification approaches with $k < k_{min}$.

The conditions presented above allow us to assess the sensitivity of poverty orderings to the choice of the social poverty measure. However, *these conditions hold only for a particular choice of dimensional weights*. With alternative selection of weights the conditions would need to be evaluated again as the values of the poverty statistics H and M depend on the specific values of the multidimensional poverty cut-off and weights.

We propose new dominance conditions to examine the robustness of poverty orderings to the choice of weighting schemes. First, we present the necessary and sufficient conditions whose fulfilment guarantees, *separately*, the robustness of conditions (1) and (2) to *any possible choice* of dimensional weights. Then, we present a sufficient condition whose fulfilment guarantees the robustness of condition (1), as well as the robustness of condition (2) by implication, to any possible choice of dimensional weights. Finally, we present a set of conditions whose fulfilment is necessary (but insufficient) to guarantee the robustness of poverty orderings to changes in the poverty index, identification cut-off, and dimensional weights. The advantage of both the exclusively sufficient and the exclusively necessary conditions resides in their easier implementation for testing purposes vis-a-vis the jointly necessary and sufficient conditions. Before presenting the new dominance results, the next subsection introduces additional notation necessary for the derivation of the conditions.

3.1 Additional Notation and Useful Poverty Statistics

We denote by $S(D)$ the power set with all possible combinations of welfare dimensions D excluding the empty set. For a given number of dimensions, D, the number of elements in $S(D)$ is equal to $2^D - 1$. Let O_s denote the population subgroup deprived *only* in dimensions $s \in S(D)$ and let c^s denote the poverty score for those deprived in the dimensions in s. Thus, for instance, for $D = 3$, the sets O_1, $O_{1,2}$, and $O_{1,2,3}$ include, respectively, the persons deprived only in dimension 1, those deprived in dimensions 1 and 2 but not in dimension 3, and those deprived in the three dimensions.

For each O_s we define the *subset headcount*, H_s, as the proportion of people who are deprived *only* in the subset of dimensions $s \in S(D)$. For any $s \in S(D)$, the measure H_s is equal to:

$$H_s \equiv \frac{|O_s|}{N}, \tag{4}$$

where $|O_s|$ is the number of people deprived exclusively in dimensions $s \in S(D)$.

Note that when the whole set of dimensions is considered, the subset headcount $H_{1,2,\dots,D}$ is basically the proportion of people who are deprived in each and every possible dimension.

We denote by Γ the set with all plausible sets of the multidimensionally poor consistent with the identification rule ρ. The set Γ includes all combinations of elements O_s with $s \in S(D)$ that could make up the set of the multidimensionally poor. Any set $\gamma \in \Gamma$ can be expressed as the union of groups O_s. For instance, in the case of $D = 2$, the set of potential poverty sets is given by $\Gamma = \{(O_{1,2}), (O_{1,2} \cup O_1), (O_{1,2} \cup O_2), (O_{1,2} \cup O_1 \cup O_2)\}$, where the first and last elements in this set correspond to the cases where the group identified as multidimensionally poor includes those deprived in all the dimensions and those deprived in any dimension, respectively. In practice the set identified as poor will depend on the threshold k and the score c^s of the different groups O_s. Note, however, that because ρ satisfies the property of poverty consistency, then any $\gamma \in \Gamma$ must include the group of those deprived in all dimensions. We denote by $\Pi(\gamma)$ the measure of any set $\gamma \in \Gamma$. This measure is defined as the proportion of the population belonging in γ which can be expressed as follows:

$$\Pi(\gamma) = \frac{1}{N} \sum_{O_s \subset \gamma} |O_s| = \sum_{O_s \subset \gamma} H_s, \tag{5}$$

where $|O_s|$ is the size of group O_s including all those deprived in the set of dimensions $s \in S(D)$. For any $\gamma \in \Gamma$, the measure $\Pi(\gamma)$ can be expressed as the sum of the subset headcounts of the sets O_s included in γ.

For any $\gamma \in \Gamma$, let $\gamma_d \subset \gamma$ denote the subset of elements of γ involving only groups deprived in dimension d. For instance, for $D = 2$, the sets γ_1 and γ_2 associated to $\gamma = (O_{1,2} \cup O_1 \cup O_2)$ are given by $\gamma_1 = (O_{1,2} \cup O_1)$ and $\gamma_2 = (O_{1,2} \cup O_2)$. For $\gamma = (O_{1,2} \cup O_1)$ the sets are $\gamma_1 = (O_{1,2} \cup O_1)$ and $\gamma_2 = (O_{1,2})$. For any $\gamma \in \Gamma$ it is easy to show that $\gamma = \bigcup_{d=1}^{D} \gamma_d$. Let Γ_d denote the set of all γ_d that can be part of a multidimensional poverty set γ. In the case of $D = 2$, the sets Γ_1 and Γ_2 have only two elements and are given by $\Gamma_1 = \{(O_{1,2}), (O_{1,2} \cup O_1)\}$ and $\Gamma_2 = \{(O_{1,2}), (O_{1,2} \cup O_2)\}$. The measure of any set $\gamma_d \in \Gamma_d$ is defined as the proportion of the population belonging in γ_d which is given by the following expression:

$$\Pi(\gamma_d) = \frac{1}{N} \sum_{O_s \subset \gamma_d} |O_s| = \sum_{O_s \subset \gamma_d} H_s, \tag{6}$$

where $|O_s|$ is again the size of group O_s including those deprived in the set of dimensions $s \in S(D)$. It is important to note that for any number of dimensions D, it holds that $\sum_{d=1}^{D} dim(\Gamma_d) \le dim(\Gamma)$. The sets Γ and Γ_d will play a key role in the new dominance conditions and they will be discussed in detail in the next subsection.

3.2 Necessary and sufficient conditions

The following condition is both necessary and sufficient to guarantee unambiguous poverty orderings within the class of measures $\mathbb{P}_1$:

Condition 3. *Consider the class of poverty measures $\mathbb{P}_1$. The following three statements are equivalent:*

1. $P(A;W,k) < P(B;W,k)$ *for all* $P \in \mathbb{P}_1$ *for any weighting vector, W, and poverty threshold, k.*

2. $H(A;W,k) \leq H(B;W,k) \quad \forall k \in [0, v_2, ..., 1] \wedge \exists k | H(A;W,k) < H(B;W,k)$, *for any weighting vector W.*

3. $\Pi^A(\gamma) \leq \Pi^B(\gamma)\ \forall \gamma \in \Gamma \wedge \exists \gamma | \Pi^A(\gamma) < \Pi^B(\gamma)$.

Proof. The equivalence between (1) and (2) follows immediately from condition (1). In order to complete the proof we just need to demonstrate the equivalence of (3) with one of the first two statements. The multidimensional poverty headcount, $H(k)$, can be expressed in terms of the size of groups O_s as follows:

$$H(Y;W,k) = \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n \geq k) = \frac{1}{N}\sum_{s \in S(D)}\mathbb{I}(c^s \geq k)|O_s|, \tag{7}$$

where $|O_s|$ is the size of group O_s including all those deprived in the set of dimensions $s \in S(D)$. The term on the right-hand side is just the observed relative frequency of the poverty set $\gamma \in \Gamma$ associated to a particular cut-off, k, and weighting vector W. Therefore, given that $H(A;W,k) \leq H(B;W,k)$ is true for any possible combination of cut-offs and weighting vectors (with at least one strict inequality), then this implies that the probability of $\Pi(\gamma)$ in A must not be greater than in B for any $\gamma \in \Gamma$ (and at least once strictly lower). On the other hand, if $\Pi(\gamma)$ for any conceivable poverty set is not greater in A than in B (and at least once strictly lower), then it must be true that $H(A;W,k) \leq H(B;W,k)$ for any possible combination of k and W (with at least one strict inequality). □

The following result establishes the necessary and sufficient conditions for unambiguous poverty orderings within the class $\mathbb{P}_2$:

Condition 4. *Consider the class of poverty measures $\mathbb{P}_2$. The following three statements are equivalent:*

1. *$P(A;W,k) < P(B;W,k)$ for all $P \in \mathbb{P}_2$ for any weighting vector, W, and poverty threshold, k.*

2. *$M(A;W,k) \leq M(B;W,k) \quad \forall k \in [0, v_2, ..., 1] \wedge \exists k | M(A;W,k) < M(B;W,k)$, for any weighting vector W.*

3. *For all $\Pi^A(\gamma_d) \leq \Pi^B(\gamma_d) \quad \forall \gamma_d \in \Gamma_d, \quad d = 1, ..., D \quad \wedge \exists \gamma_d | \Pi^A(\gamma_d) < \Pi^B(\gamma_d)$.*

Proof. The equivalence between (1) and (2) follows immediately from condition (2). To prove the equivalence with (3) it is important to note first that, for a given combination of weights and multidimensional cut-off, the adjusted headcount ratio, $M(k)$, can be expressed as the weighted sum of the probabilities of the sets γ_d included in the set of multidimensionally poor γ associated to that particular combination of k and W:

$$M(Y;W,k) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{I}(c_n \geq k) c_n = \sum_{d=1}^{D} w_d \Pi(\gamma_d), \tag{8}$$

The difference in adjusted headcount ratios can then be expressed as:

$$M(A;W,k) - M(B;W,k) = \sum_{d=1}^{D} w_d [\Pi^A(\gamma_d) - \Pi^B(\gamma_d)]. \tag{9}$$

Since $[\Pi^A(\gamma_d) - \Pi^B(\gamma_d)] \leq 0$ for all $\gamma_d \in \Gamma_d$ and $d = 1, ..., D$ (with at least one strict inequality), then for any vector, W, and cut-off, k, it is true that $[M(A;W,k) - M(B;W,k)] \leq 0$ (with at least one strict inequality) which proves the sufficiency part of the equivalence. Now assume that for some γ_d it holds that $[\Pi^A(\gamma_d) - \Pi^B(\gamma_d)] > 0$, then it is possible to find a vector of dimensional weights, W, such that $[M(A;W,k) - M(B;W,k)] > 0$. But this contradicts statement (2). Therefore it must be true that $\Pi(\gamma_d)$ in A is not greater than in B for all $\gamma_d \in \Gamma_d$ and $d = 1, ..., D$ (and at least once strictly lower). □

The following remark establishes the link between condition (3) and (4):

Remark 2. *If $H(A;W,k) \leq H(B;W,k) \quad \forall k \in [0,1] \quad \wedge \exists k | H(A;W,k) < H(B;W,k)$ for any vector of weights, W, then $M(A;W,k) \leq M(B;W,k) \quad \forall k \in [0, v_2, ..., 1] \quad \wedge \exists k | M(A;W,k) < M(B;W,k)$ for any weighting vector, W.*

Proof. This remark is an extension of remark (1) to any possible vector of weights and its proof follows from Alkire and Foster (2011, Theorem 2). □

Remark (2) implies the existence of dominance for the class of poverty measures $\mathbb{P}_2 \subset \mathbb{P}_1$ whenever there exists dominance within the class $\mathbb{P}_1$ of poverty measures.

3.3 General sufficient conditions

Conditions (3) and (4) provide a simple way to ascertain the existence of dominance in poverty comparisons based on counting measures. However, testing those conditions may require comparing a large number of statistics. In fact, as we show in the next section, the number of elements in the sets Γ and Γ_d increases exponentially with the number of dimensions involved in the poverty comparisons. With that concern in mind, we derive a set of useful conditions which are much easier to implement in practice, especially when D is relatively large, as they require a much smaller number of statistics. Firstly, we derive a sufficient condition whose fulfillment guarantees a robust pairwise poverty ordering for any poverty measures in the most general classes $\mathbb{P}_1$ and $\mathbb{P}_2$, as well as, the measures H and M. Secondly, in the next subsection, we introduce two necessary conditions whose violation implies that no unambiguous poverty ordering can be established when comparing two populations. The sufficient condition is the following:

Condition 5. *Let $P(Y;W,k)$ be any poverty measure belonging to the class $\mathbb{P}_1$. If all the subset headcounts H_s of A are never higher than those of B and at least one of them is strictly lower, then $P(A;W,k) \leq P(B;W,k)\ \forall k \wedge \exists k | P(A;W,k) < P(B;W,k)$ for all possible weighting vectors, W.*

Proof. From equation (5) we know that, for any $\gamma \in \Gamma$, the measure $\Pi(\gamma)$ can be expressed as a sum of subset headcounts. Therefore if all the subset headcounts of A are never higher than those of B and at least one of them is strictly lower, then the value of $\Pi(\gamma)$ in A will never be higher than that in B for any $\gamma \in \Gamma$ (and at least one will be strictly lower).[9] From condition (3) this implies that $P(A;W,k) \leq P(B;W,k)\ \forall k \wedge \exists k | P(A;W,k) < P(B;W,k)$ for all possible weighting vectors, W, and all $P(Y;W,k) \in \mathbb{P}_1$. □

Note this sufficient condition applies also to the class $\mathbb{P}_2$ and the indices H and M. This is because, by conditions (3) and (4), dominance within the class $\mathbb{P}_1$ implies dominance within the class $\mathbb{P}_2$, as well as the poverty indices H and M. Being a sufficient condition, a violation of (5) does not rule out poverty dominance of A over B. However, as shown in the necessary condition (6) below, if condition (5) is violated because $H^A_{(1,2,\ldots,D)} > H^B_{(1,2,\ldots,D)}$, then we can actually conclude that A does not dominate B. Hence a combination of condition (5) and the necessary conditions of the next section, can go a long way in ascertaining pairwise poverty dominance (or lack thereof) when D is large.

[9] But note that reverse is not true.

3.4 General necessary conditions

We derive two useful necessary conditions which are easy to implement, as they require one and D statistics, respectively. The first of these necessary conditions is the following:

Condition 6. *Let $P(Y;W,k)$ be any poverty measure belonging to the classes $\mathbb{P}_1$ or $\mathbb{P}_2$, or the multidimensional measures, $H(Y;W,k)$ and $M(Y;W,k)$. If $P(A;W,k) \leq P(B;W,k)\ \forall k \wedge \exists k | P(A;W,k) < P(B;W,k)$ for all possible weighting vectors, W, then: $H^A_{(1,2,...,D)} \leq H^B_{(1,2,...,D)}$.*

Proof. First note that the set $O_{1,2,...,D}$ including all those individuals deprived in all dimensions belongs to any multidimensional poverty set in Γ and also to the sets Γ_d with $d = 1,...,D$. From conditions (3) and (4) we know that, when $P(A;W,k) < P(B;W,k)$ for all W and k, the relative frequency of all elements of Γ and Γ_d in A must not be greater than in B, which implies that $\Pi^A(O_{1,2,...,D}) = H^A_{(1,2,...,D)} \leq H^B_{(1,2,...,D)} = \Pi^B(O_{1,2,...,D})$.

□

Condition (6) states that whenever multidimensional poverty in population A is lower than in population B for every possible weighting vector, W, and identification cut-off, k, then it must be the case that the percentage of people deprived in every dimension in A (i.e. following an intersection approach to poverty identification) cannot be higher than the percentage of people from B in the same situation. This is a simple but powerful condition: it basically means that we can rule out the possibility of dominance between two populations by simply comparing the percentage of people deprived in all dimensions in each population. Note that this condition applies to any poverty index in $\mathbb{P}_1$ or $\mathbb{P}_2$, as well as to the multidimensional headcount (H; not included in class $\mathbb{P}_1$) and the adjusted headcount ratio (M; not included in class $\mathbb{P}_2$).

The second necessary condition is:

Condition 7. *Let P be any poverty measure belonging to the classes $\mathbb{P}_1$ or $\mathbb{P}_2$, or the multidimensional measures, H and M. If $P(A;W,k) \leq P(B;W,k)\ \forall k \wedge \exists k | P(A;W,k) < P(B;W,k)$ for all possible weighting vectors, W, then: $U_d(A) \leq U_d(B)\ \forall d \in [1,2,...,D]$.*

Proof. Note that for all $d \in [1,2,...,D]$, it is easy to show that the set including all those individuals deprived in dimension d always belongs to the sets Γ and Γ_d. From conditions (3) and (4) we know that, when $P(A;W,k) < P(B;W,k)$ for all W and k, the relative frequency of all elements in Γ and Γ_d in A must not be greater than in B, which implies $U_d(A) \leq U_d(B)\ \forall d \in [1,2,...,D]$.

□

Condition (7) states that if poverty in population A is unambiguously lower than in B then it must be the case that all the uncensored deprivation headcount ratios in A cannot be higher than their respective counterparts from B. This is, again, a simple but powerful condition: without comparing the relative frequencies of all elements in the sets Γ and Γ_d, if there exists just one variable d for which $U_d(A) > U_d(B)$, then we can rule out the possibility that A dominates B for every poverty measure in $\mathbb{P}_1$ or $\mathbb{P}_2$, and any conceivable weighting vector, W, and cut-off value, k.

4 Application of the New Dominance Conditions

The dominance results presented in the previous section provide a useful analytical framework to evaluate the robustness of poverty orderings based on counting measures. The evaluation of those conditions, however, requires the computation and comparison of a number of statistics which grows with the number of welfare dimensions. Table 1 below shows the number of statistics involved in each condition for values of D from 2 to 5.

In general, conditions (1) and (2) involve a small number of statistics vis-a-vis the conditions applicable to the case of variable weights, i.e. (3) and (4). This is not surprising as these conditions permit to assert poverty dominance within the classes $\mathbb{P}_1$ and $\mathbb{P}_2$ only for *a given vector of dimensional weights*. Thus, for any vector of weights, conditions (1) and (2) require the comparison of the indices $H(Y;W,k)$ and $M(Y;W,k)$ for all relevant values of the threshold k. These values depend on the specific vector of weights and it is easy to show that the number of relevant values is never greater than $\sum_{i=0}^{D}\binom{D}{i} = 2^D$.

The necessary and sufficient conditions (3) and (4) are the most demanding of all conditions since they require the comparison of all the sets γ and γ_d belonging to the sets Γ and Γ_d. While derivation of these sets is trivial when D is small, it gets more complex as the number of dimensions increases. This is because the number of elements in Γ and Γ_d grows fast with D as the combinations of groups O_s that can make up the set of the multidimensionally poor rise exponentially with the number of dimensions.

In order to derive the sets Γ and Γ_d we developed two search algorithms that identify the combinations of O_s that can form any plausible poverty set γ and their dimensional components γ_d.[10] The key to the identification of the potential poverty

[10]The algorithms *gamma* and *gammad* are coded in Stata version 14.0 and are included as part of the Stata package *Domcount* especifically developed to empirically implement the new dominance conditions. The package is available at `https://drive.google.com/file/d/0B4MaiGQpsjKqeUlvQWJhUWpJbk0/view`.

sets in these algorithms is the consistency property of the poverty identification function ρ_k (Lasso de la Vega, 2010). This property requires that, for any two sets O_s and $O_{s'}$ with $c_s \leq c_{s'}$, if the set O_s belongs to a given poverty set γ then that must be the case also of set $O_{s'}$. Thus, for instance, if a poverty set γ includes the group O_1 comprising those deprived only in dimension 1, then it must also include all those sets O_s with larger $c_{s'}$ involving combinations of deprivation in dimension 1 and any other dimensions. For instance, in the case of $D = 3$, if O_1 belongs to any set γ then that must be the case also of the groups $O_{1,2}$, $O_{1,3}$, and $O_{1,2,3}$, including those deprived in dimensions 1 and 2; 1 and 3; and 1, 2, and 3; respectively. As Table (1) shows, the number of potential poverty sets grows more than exponentially for conditions (3) and (4); with the number of dimensions jumping from 18 when $D = 3$ to 7,579 when $D = 5$, in the case of condition (3). Although smaller, the number of sets γ_d required to evaluate condition (4) also grows significantly fast, with D reaching 690 when $D = 5$.

Table 1 – Number of statistics involved in each dominance condition

	Statistic	$D = 2$	$D = 3$	$D = 4$	$D = 5$
Fixed weights					
Necessary & sufficient					
Condition 1	$H(Y;W,k)$	4	8	16	32
Condition 2	$M(Y;W,k)$	4	8	16	32
Variable weights					
Necessary & sufficient					
Condition 3	$\Pi(\gamma)$	4	18	166	7579
Condition 4	$\Pi(\gamma_d)$	3	10	63	690
Sufficient					
Condition 5	H_s	3	7	15	31
Necessary					
Condition 6	$H_{(1,2,\ldots,D)}$	1	1	1	1
Condition 7	$U_d(Y)$	2	3	4	5

The sufficient condition (5) involves the comparison of the subset headcounts H_s for all combinations of dimensions s in the power set $S(D)$. The number of elements in this set, excluding the empty set, is equal to $2^D - 1$ which gives the number of statistics to be compared. Finally the necessary conditions (6) and (7) are the easiest to evaluate as they require, respectively, the comparison of the percentage of people deprived in all dimensions, and the uncensored deprivation headcounts U_d reporting the proportion of people deprived in each of the dimensions.

4.1 A general testing framework

While the conditions presented above can be tested with many different tests, we propose an intersection-union, multiple-comparison test which is convenient for its simplicity, generally low size, and decent power for pair-wise population comparisons (Dardanoni and Forcina, 1999; Hasler, 2007). Evaluating each of the dominance conditions requires computing and comparing $R \geq 1$ sample statistics in the forms of sample means, e.g. $M(k)$ for all relevant values of k, which are asymptotically standard-normally distributed (i.e. the assumptions of the central limit theorem hold).

Let $z(r) \equiv \frac{X^A(r)-X^B(r)}{SE[X^A(r)-X^B(r)]}$, where $X^A(r)$ is a sample mean for A (e.g. $M^A(1)$) and $SE[X^A(r) - X^B(r)]$ is the standard error of the difference $X^A(r) - X^B(r)$. We propose the following null and alternative hypotheses:

$$Ho : z(r) = 0 \ \forall r = 1, 2, ..., R$$

$$Ha : z(r) < 0 \ \forall r = 1, 2, ..., R$$

When testing these hypotheses we reject the null in favour of the alternative if $\max\{z(1), z(2), ..., z(R)\} < z_\alpha < 0$, where z_α is a left-tail critical value, and α is both the size of a single-comparison test as well as the overall level of significance of the multiple-comparison test. It is not difficult to show that, generally, the overall size of the test will be lower than α. Given the nature of the conditions, if we reject the null in favour of the alternative hypothesis then A dominates B in the sense of being deemed less poor for a broad class of poverty measurement choices (which depends on the condition in question).

The formula of the specific z-statistics varies across conditions as different conditions look at different aspects of the distribution of deprivations. Below we present the statistics used for each condition.

Test of conditions 1 and 2

For condition (1) we use z-statistics of the form:

$$z(k) = \frac{H^A(k) - H^B(k)}{\sqrt{\frac{\sigma^2_{H^A}(k)}{N^A} + \frac{\sigma^2_{H^B}(k)}{N^B}}}, \tag{10}$$

where:

$$\sigma^2_{H^A}(k) = H^A(k)[1 - H^A(k)]. \tag{11}$$

For condition (2) we use the same statistic but replacing $H(k)$ with $M(k)$, and noting that the variance in this case is given by:

$$\sigma^2_{M^A}(k) = \frac{1}{N^A} \sum_{n=1}^{N^A} [c_n]^2 \mathbb{I}(c_n \geq k) - [M^A(k)]^2 \tag{12}$$

Test of conditions 3 and 4

These conditions require the comparison of the measure of the sets $\gamma \in \Gamma$ and $\gamma_d \in \Gamma_d$. To this purpose, for condition (3) we consider statistics of the form:

$$z(\gamma) = \frac{\Pi^A(\gamma) - \Pi^B(\gamma)}{\sqrt{\frac{\sigma^2_{\Pi^A(\gamma)}}{N^A} + \frac{\sigma^2_{\Pi^B(\gamma)}}{N^B}}}, \tag{13}$$

where $\Pi(\gamma)$ is given by expression (5) and:

$$\sigma^2_{\Pi^A(\gamma)} = \Pi^A(\gamma)[1 - \Pi^A(\gamma)]. \tag{14}$$

For condition (4) the formulae are the same but simply replacing $\Pi(\gamma)$ with $\Pi(\gamma_d)$.

Test of condition 5

This condition compares the subset headcounts H_s for all combinations of dimensions s in the power set $S(D)$. We use the following statistic:

$$z(s) = \frac{H^A_s - H^B_s}{\sqrt{\frac{\sigma^2_{H^A_s}}{N^A} + \frac{\sigma^2_{H^B_s}}{N^B}}}, \tag{15}$$

where H_s is given by equation (4) and:

$$\sigma^2_{H^A_s} = H^A_s[1 - H^A_s]. \tag{16}$$

Test of condition 6 and 7

For the necessary condition (7) we use z-statistics of the form:

$$z_d = \frac{U_d(A) - U_d(B)}{\sqrt{\frac{\sigma^2_{U_d(A)}}{N^A} + \frac{\sigma^2_{U_d(B)}}{N^B}}}, \tag{17}$$

where:

$$\sigma^2_{U_d(A)} \equiv U_d(A)[1 - U_d(A)]. \tag{18}$$

These formulae can also be used for condition (6) but noting that evaluating this condition requires only the comparison of the percentage of people deprived in all possible dimensions which is given by $H_{1,2,...,D}$.

5 Empirical illustration: Poverty in Australia in the 2000s

We use the new dominance results to evaluate the robustness of poverty trends in Australia over the first decade of the XXI century. This was a period of strong income growth in which Australia outperformed most developed countries. This was particularly true during the period 2001-2007, where incomes grew at an average rate above 3 per cent largely driven by the mining boom and favourable trends in commodity prices. Although to a lesser extend than the US and European countries, Australia's economic performance was also affected by the Global Financial Crisis (GFC) as reflected in the rapid increase in unemployment between April 2008 and June 2009 (from 4.1 to 5.7 per cent). This negative shock, together with the declining mining boom, led to slower income and employment growth in the period 2008-2010 relative to the pre-GFC years.

We evaluate poverty trends in Australia using data from the Household Income and Labour Dynamics in Australia (HILDA) survey. This is a nationally representative survey initiated in 2001, which collects detailed socio-economic information from more than 7,000 households and their members every year. For the illustration we consider three indicators of economic disadvantage: a binary income poverty indicator equal to 1 if the household's annual income is below 60 per cent of the median equivalent income; an asset-poverty indicator which is equal to 1 when the household lacks enough assets to sustain its members above the income poverty line for three months; and a measure of financial hardship equal to 1 whenever the household reports that at least three of the following circumstances occurred along the financial year: could not pay electricity, gas or telephone bills on time; could not pay the mortgage or rent on time; pawned or sold something; went without meals; was unable to heat the home; asked for financial help from family, friends, or community organizations. For the income and wealth poverty indicators, the income and wealth variables were adjusted by household size using the OECD modified equivalence scale that assigns a value of 1 to the first adult, 0.5 to subsequent adults in the household, and 0.3 to every member under the age of 15. The unit of analysis for poverty comparisons is the individual and each individual is assigned the value of the poverty indicators computed at the household level.

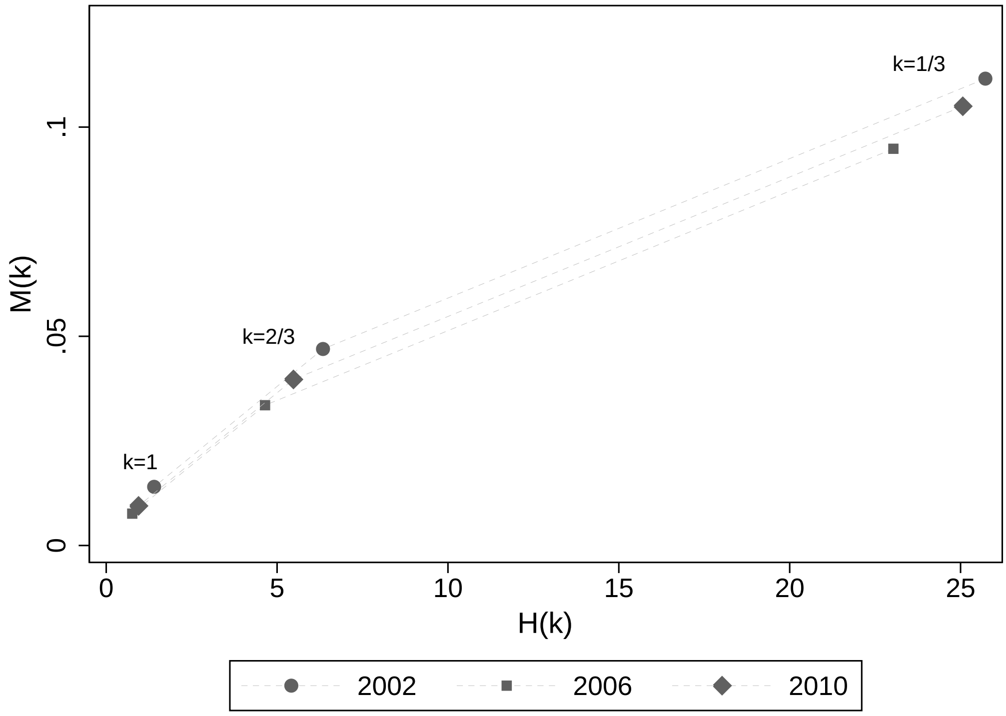
Table 2 shows the prevalence of the poverty indicators for the years 2002, 2006, and 2010. The levels of economic deprivation declined substantially during the years of strong economic growth that preceded the GFC. Income and wealth poverty rates fell, respectively, about 2 and 1.4 percentage points from 2002 to 2006. The income and wealth gains led to a decline in the proportion of people experiencing financial hardship which, by 2006, was more than 1.7 percentage points below that in 2002. By contrast, economic disadvantage increased in the years following the GFC. By 2010 the income and wealth poverty rates were above those of 2006 but still below the levels observed at the start of the decade.

Table 2 – Poverty indicators in Australia(%)

Year	Income	Wealth	Financial hardship
2002	18.47	8.36	6.65
2006	16.55	6.97	4.93
2010	18.21	7.39	5.89

Figure 1 shows the multidimensional headcount $H(k)$ (horizontal axis) and the adjusted headcount ratio $M(k)$ (vertical axis) for the years 2002, 2006, and 2010 assuming equal weights for the three dimensions. Estimates of the indices are displayed for each relevant values of the poverty threshold k (1, $2/3$, and $1/3$, from the origin outward). A point in the graph thus represents the vector (H, M) for a year and poverty cut-off, such that, for a given value of k, points located further away from the origin indicate higher levels of multidimensional poverty. Inspection of the figure reveals a substantial decline in poverty in the years preceding the GFC. Our estimates of M and H for 2002 are larger than those for 2006 for any relevant value of k. This positive trend was partially reversed in the years following the GFC. Indeed, poverty estimates for 2010 are greater or equal than those in 2006 for any poverty cut-off. Despite this change, poverty levels by 2010 were still lower than those at the start of the decade.

Figure 1 – M(k) and H(k) indices (equal weights)



To evaluate whether the poverty orderings based on the $H(k)$ and $M(k)$ indices for the case of equal weights coincide with those of any measure in the classes $\mathbb{P}_1$ and $\mathbb{P}_2$ we apply the dominance conditions (1) and (2). Tables 3 and 4 present the statistics required to test each of those conditions.[11] For this and all subsequent tests, we present the results for all pairwise comparisons such that the statistic in each cell serves to test whether the year in the column dominates (i.e., has less poverty than) the year in the row. For conditions (1) and (2), the statistics in the tables correspond to the maximum value of the $z(k)$ statistics ($k = 1, 2/3, 1/3$) relevant for each pairwise comparison.[12] When comparing 2002 with 2006, we find statistical evidence to reject the hypothesis of equal poverty in favour of the alternative whereby poverty declined between the two years. Thus, under the assumption of equal weights, using standard significance levels we can conclude that poverty in 2006 was lower than in 2002 for any poverty index in the class $\mathbb{P}_1$. Based on our estimates, we cannot unambiguously assert that poverty levels

[11]These statistics, as well as those used to test the other conditions, were computed using the Stata program *robust* included in the Stata package *Domcount* available at `https://drive.google.com/file/d/0B4MaiGQpsjKqeUlvQWJhUWpJbk0/view`.

[12]Note that the statistics in the column for 2006 are the same for conditions (1) and (2). This is because, for the statistics based on both the M and H measures, the maximum difference between 2006 and the other two years occurs for $k = 1$, and we know that $H(1) = M(1)$.

in 2010 were different to those in 2006. However, the results for 2010 and 2002 show that the level of poverty in 2010 was still below that in 2002, although this results holds only for the class $\mathbb{P}_2$ of poverty measures as we fail to reject the null hypothesis for condition (1).

Table 3 – Test of condition 1 (maximum statistics)

$Ho: H(t_A;k) = H(t_B;k)\forall k$ versus $Ha: H(t_A;k) < H(t_B;k)\forall k$

t_B \ t_A	2002	2006	2010
2002	0.00	-4.70	-1.14
2006	5.64	0.00	3.54
2010	3.19	-1.49	0.00

Table 4 – Test of condition 2 (maximum statistics)

$Ho: M(t_A;k) = M(t_B;k)\forall k$ versus $Ha: M(t_A;k) < M(t_B;k)\forall k$

t_B \ t_A	2002	2006	2010
2002	0.00	-4.70	-2.41
2006	6.30	0.00	3.86
2010	3.19	-1.49	0.00

These dominance results apply only to the case of equal weights. Nothing a priori ensures that they will hold under different weighting schemes. Can we unambiguously claim that poverty in 2006, or 2010, was lower than in 2002 regardless of the choice of dimensional weights? In order to answer this question we now turn to the new poverty dominance conditions.

We start the analysis looking at the necessary conditions as these allow us to rule out the existence of dominance by checking only a limited number of conditions. Table 5 shows the statistics to test the necessary condition (7) which involves the comparison of the uncensored deprivation headcount U_d of the different dimensions. The value reported in each cell corresponds to the maximum value of the z_d statistics ($d = 1, 2, 3$) relevant for each pairwise comparison. A sufficiently large negative value of the statistic is taken as evidence against the null hypothesis and the failure to reject this hypothesis means that we can rule out the existence of dominance between the compared years. Interestingly, our results rule out the existence of dominance for all pairwise comparisons except that between 2006 and 2002. Thus, we cannot establish any unanimous ranking for any of the poverty comparisons involving 2002 versus 2010 and 2006 versus 2010. *This result illustrates the sensitivity of multidimensional poverty orderings based on counting poverty measures to the choice of dimensional weights and poverty cut-off.*

In order to evaluate whether poverty in 2006 was unambiguously lower than in 2002 we use the necessary and sufficient conditions. Table 6 shows the statistics required to test the sufficient condition (5). This condition involves the comparison of the subset headcounts and the rejection of the null hypothesis implies that the sufficient condition for dominance is satisfied. Using standard levels of significance, we find no statistically significant evidence to reject the null in any pairwise comparison. In particular, the value of the statistic for the comparison of 2006 against 2002 is 0.07, which implies that the dominance of 2006 over 2002 cannot be unambiguously asserted using the sufficient condition. However, this result does not rule out the possibility of dominance, since condition (5) is sufficient but not necessary.

Table 7 shows the statistics to evaluate the necessary and sufficient condition (3). Evaluating this condition requires the comparison of the measure of all poverty sets $\gamma \in \Gamma$. Interestingly, the result for the comparison of 2006 and 2002 suggests there is enough evidence to reject the null and therefore to assert that poverty by 2006 was unambiguously lower than in 2002 for any choice of dimensional weights and poverty cut-off and any poverty measure in $\mathbb{P}_1$ or $\mathbb{P}_2$.

Table 5 – Test of necessary condition 7 (maximum statistics)

$Ho: U_d(t_A) = U_d(t_B)\ \forall d \in [1, 2, ..., D]$ versus $Ha: U_d(t_A) < U_d(t_B) \forall d$

t_B \ t_A	2002	2006	2010
2002	0.00	-3.82	-0.50
2006	5.60	0.00	3.26
2010	2.69	-1.22	0.00

Table 6 – Test of sufficient condition 5 (maximum statistics)

$Ho: H_s(t_A) = H_s(t_B)\ \forall s \in S(D)$ versus $Ha: H_s(t_A) < H_s(t_B)\ \forall s \in S(D)$

t_B \ t_A	2002	2006	2010
2002	0.00	0.07	1.05
2006	4.70	0.00	3.33
2010	3.19	0.29	0.00

Table 7 – Test of necessary and sufficient condition 3 (maximum statistics)

$Ho : \Pi^{t_A}(\gamma) = \Pi^{t_B}(\gamma)\ \forall\gamma \in \Gamma$ versus $Ha : \Pi^{t_A}(\gamma) < \Pi^{t_B}(\gamma)\ \forall\gamma \in \Gamma$

t_B \ t_A	2002	2006	2010
2002	0.00	-3.45	-0.50
2006	6.79	0.00	4.27
2010	3.78	-0.50	0.00

6 Concluding remarks

In this paper we sought to derive robustness conditions to evaluate the sensitivity of poverty orderings based on counting measures. Building on the results in Lasso de la Vega (2010), we propose fundamental conditions whose fulfilment is both necessary and sufficient to ensure that both first-order and second-order propositions work for any conceivable weighting vector with positive elements. However, since these conditions may be cumbersome to implement when the number of variables is large,[13] we also derived two useful conditions whose fulfillment is necessary, but insufficient, for robust first- and second-order comparisons using any possible weighting vector. While these conditions are insufficient, they are fewer in number, and much easier to compute. When they are not met we can immediately rule out the robustness of second-order dominance in poverty reduction to any choice of weights. We also provided a useful sufficient condition whose fulfillment guarantees first and second-order comparisons for any possible weighting vector. Though this condition is not necessary (hence its violation would not preclude the existence of a dominance relationship), it also bears the advantage of a much easier implementation vis-a-vis the set of necessary and sufficient conditions.

Above and beyond the conditions derived in this paper, it is also possible to derive sets of necessary and sufficient conditions which guarantee robust egalitarian poverty comparisons for a *subset of weights*, as well as for broader sets of weights (e.g. admitting zero values). Likewise further useful necessary conditions are derivable if we opt to restrict the set of admissible weighting vectors, or the domain of k cut-offs, or both jointly. Some examples are available upon request. The development of general methods for the derivation of conditions whose fulfillment guarantees *partial robustness*, i.e. full robustness only to combinations of subsets of parameters (e.g. joint restrictions on weights and cut-offs, etc.) is left for future research.

[13]Even though we have also rendered a ready-to-use algorithm available for Stata users.

Finally, the empirical application to Australia over time illustrated the usefulness of the new robustness conditions. In the Australian case, we learned that poverty reduction between 2002 and 2006 was robust to any counting poverty functional form, any poverty cut-off k, and any vector of deprivation weights. By contrast, the apparent trend of poverty increasing from 2006 to 2010 but leading to overall lower poverty levels compared to 2002, did not prove robust to any conceivable combination of the aforementioned parameters.

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