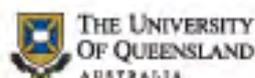




How to Measure Inequality of Opportunity: A Hands-On Guide

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NON-TECHNICAL SUMMARY

The measurement of inequality of opportunity is a growing topic in economics. In recent years many theoretical papers have been published and the number of empirical contributions to this literature has exploded in the last two decades. The increasing quality and availability of survey data will likely encourage further empirical research. Not surprisingly, journal articles and volume chapters have recently reviewed the main approaches to measuring inequality of opportunity and discussed the existing evidence. The aim of this paper is complementary to those contributions: it discusses the many practical issues that typically arise when measuring inequality of opportunity with survey data.

The paper originates from lecture notes for the workshop “Measuring inequality of opportunity” held at ISSR, The University of Queensland in February 2016. The workshop aimed at covering the main issues of the applied literature on inequality of opportunity measurement. The material is presented in an attempt to attract the interest of a multidisciplinary audience of social scientists and therefore does not get into the details of some more technical aspects. For example, the discussion of inference (generally based on bootstrap) is absent. The comprehensive list of references provided may accommodate the needs of most demanding readers.

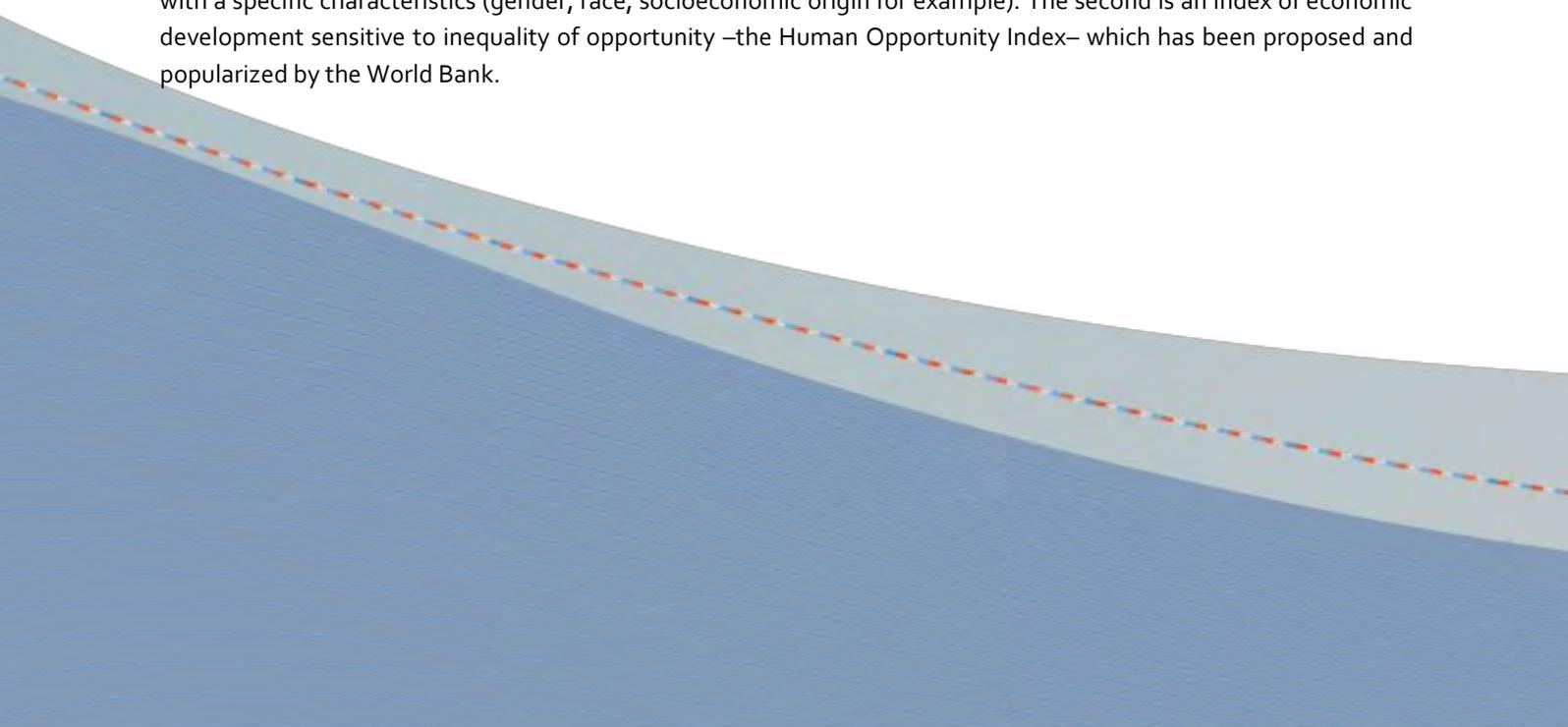
The paper is organised in five sections. The first section introduces the ideal of equality of opportunity as it developed in the economic literature in the last decades.

The second section discusses two approaches to measure inequality of opportunity. The first proposed by John Roemer and the second introduced by Marc Fleurbaey and Francois Maniquet. These two approaches quantify inequality of opportunity following a similar three-step method: i) define the properties that a distribution of valuable outcome –such as income or health– must have to satisfy the principle of equality of opportunity; ii) obtain a counterfactual distribution of which reflects the violations of those properties in the actual distribution; iii) measure inequality in the counterfactual distribution of unfair inequality.

The third section discusses alternative methods to measure inequality of opportunity. The literature has proposed two main approaches –a parametric and a non-parametric approach– both methods have advantages and shortcomings that are discussed in the section.

The fourth section suggests two inequality measures that can be used to measure inequality in the estimated counterfactual distribution and discusses the effects of different choices.

The last section is devoted to two more advanced topics. The first is a method to decompose total inequality of opportunity by sources, that is it introduces a method to identify the share of inequality of opportunity associated with a specific characteristic (gender, race, socioeconomic origin for example). The second is an index of economic development sensitive to inequality of opportunity –the Human Opportunity Index– which has been proposed and popularized by the World Bank.



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Abstract

The measurement of inequality of opportunity is a growing topic in economics. In recent years many theoretical papers have been published and the number of empirical contributions to this literature has exploded in the last two decades. The increasing quality and availability of survey data will likely encourage further empirical research. Not surprisingly, journal articles and volume chapters have recently reviewed the main approaches to measuring inequality of opportunity and discussed the existing evidence. The aim of this paper is complementary to those contributions: it discusses the many practical issues that typically arise when measuring inequality of opportunity with survey data. The paper originates from my lecture notes for the workshop “Measuring inequality of opportunity” held at ISSR, The University of Queensland in February 2016. The workshop aimed at covering the main issues of the applied literature on inequality of opportunity measurement. The material is presented in an attempt to attract the interest of a multidisciplinary audience of social scientists and therefore does not get into the details of some more technical aspects.

Keywords: inequality of opportunity; survey data; measurement; practical issues

1 The idea of equal opportunity

The large majority of politicians place equality of opportunity high on their agenda. It is possible to quote politicians with extremely different political orientations in support of equal opportunity: from Margaret Thatcher¹ to Raul Castro², from Nelson Mandela³ to Nicolas Sarkozy⁴.

There are two important reasons that explain the popularity of this political ideal. Firstly, equality of opportunity incorporates two fundamental ethical principles: equality and freedom. Secondly, equality of opportunity is very frequently so vaguely stated to be uncontroversial.

What explains the fact that the term ‘equality of opportunity’ remains vague in the public debate? Equality of opportunity merges two values: equality and freedom. Ideally, equality of opportunity is achieved when all individuals are free to choose from the same set of opportunities. However, the principle of freedom and of equality are only partly compatible. Any precise definition of equality of opportunity represents one particular way to solve the incompatibility between the two values: a way to balance between freedom and equality. The ability to balance and mitigate these two fundamental values is probably the main reason that explains the popularity of the ideal of equal opportunity. This became especially true after the end of the Cold War. On the one hand, planned economies collapsed politically and economically because of the lack of freedom and incentives. On the other, the richer Western world became aware that economic prosperity was not sufficient to guarantee equal chances of success for all, irrespective of race, sex and socioeconomic origin.

However, when the exact meaning of equality of opportunity is detailed and pragmatically defined, the result is a vast range of heterogeneous and conflicting definitions. Equal opportunity may be understood as non-discrimination, prescribing that all individuals should be treated equally. To this formal definition others oppose a substantive version of the same principle: equality of opportunity requires that all individuals have the same chances to obtain valuable outcomes. Contrary to the formal equality of opportunity, the substantive version of the principle is very likely to entail large, redistributive intervention.

¹“First, that the pursuit of equality itself is a mirage. What’s more desirable and more practicable than the pursuit of equality is the pursuit of equality of opportunity.” Speech to the Institute of SocioEconomic Studies, New York, September, 15, 1975.

²“*socialismo significa justicia social e igualdad, pero igualdad de derechos, de oportunidades, no de ingresos*”, speech at the Asamblea Nacional del Poder Popular, La Habana, July 11, 2008.

³“*I have fought against white domination, and I have fought against black domination. I have cherished the ideal of a democratic and free society in which all persons live together in harmony with equal opportunities.*”, Speech On his release addressing crowds from the balcony of Cape Town’s City Hall on February 11, 1990.

⁴“*L’égalité républicaine, c’est l’égalité devant la loi, l’égalité des droits et des devoirs, c’est l’égalité dignité des personnes, c’est l’égalité des chances.*” Speech at the l’Ecole Polytechnique, Paris, December 17, 2008.

In the economic literature since Rawls (1971), a number of authors have suggested that a society in which social positions are formally available to everyone does not guarantee equal opportunity. In Rawls' view a "fair equal of opportunity" is realised only when equally talented individuals have the same chance to obtain desirable social positions. This idea has been framed in many different versions (Dworkin, 1981a,b; Arneson, 1989; Cohen, 1989; Nozick, 1974) and has led to a number of different definitions of equality of opportunity. The majority of these authors distinguish between two types of sources of inequality: morally objectionable and morally acceptable sources. In what follows we will focus on definitions that distinguish the two type of sources on the basis of two types of personal traits: attributes for which it is morally correct to hold individuals responsible, and those circumstances beyond individual control for which individuals should not be held responsible. Inequality of opportunity arises only when, and to the extent in which, that inequality is due to difference in circumstances beyond individual control.

Following the seminal contributions by Roemer (1998) and Fleurbaey (2008), a number of measures of inequality of opportunity have been derived by different definitions of equal opportunity. A measure of inequality of opportunity is a measure of how far a given distribution of individual outcomes is from equal opportunity. Moreover, because Roemer's and Fleurbaey's measure of inequality of opportunity are easily implementable, the last two decades have witnessed an explosion of empirical contributions proposing estimates of inequality of opportunity. Inequality of opportunity has been measured in many dimensions of individual well being (income, consumption, health, education) and for a large number of countries.⁵

2 Measures of unequal opportunities

A meaningful measure of inequality of opportunity is based on a precise definition of equal opportunity. All the measures presented in these notes are based on definitions which share an important characteristic: they consider unproblematic any inequality due to responsibility characteristics, while identifying inequality of opportunity as inequality due to variables beyond individual control. These approaches therefore share the assumption that it is possible to disentangle these two types of individual characteristics. Other authors have suggested that such a distinction is impossible both in theory and in practice; an interesting discussion of this can be found in Fishkin (2014). However, none of these definitions indicate which individual characteristics belong to the first type

⁵Note that there exists a distinct literature that has proposed measures of inequality of opportunity as measures of inequality between opportunity sets. Because of its high level of abstractness, this literature has generated very few empirical developments and are beyond the scope of this paper. Interested readers can find a review of the theoretical papers on this topic in Ferreira and Peragine (2015).

of traits and which characteristics belong to the second. All theories presented in the following sections consider that it is the society in question (or the social planner) that decides which factors are to be classified as circumstances beyond individual control and which are to be considered to be personal choices.

I attempt in this essay to propose a precise way that can organise our disparate views about equal opportunity. More specifically, different people have different conceptions about where the starting gate should be, or about the degrees to which individuals should be held accountable for the outcome or advantage they eventually enjoy. My purpose is to propose an algorithm which will enable a society to translate any such views about personal accountability into a social policy that will implement a kind or degree of equal opportunity consonant with that views.

Roemer (1998) p. 2

2.1 Roemer's approach

Roemer (1998) represents the seminal contribution for the empirical literature on inequality of opportunity. In his book, Roemer did not explicitly write down a definition of inequality of opportunity. His theory proposes a criterion to select the redistributive policy that would equalize opportunity in a society. However, his theory has been translated into more than one definition of inequality of opportunity.

Roemer's theory divides factors that determine individual outcomes into two types: factors over which individuals have control, which he calls "effort", and factors for which individuals cannot be held responsible, which he calls "circumstances". He defines a situation of equal opportunity in the distribution of a certain desirable outcome – or "advantage", to use his terminology – as the situation in which individuals are compensated for the difference in their circumstances, insofar as those differences affect the advantage they attain. To implement the equality of opportunity policy, Roemer proposes to partition the population into a set of *types*. A type is a set of individuals characterised by exactly the same circumstances (gender, race, socioeconomic background,...). By exerting effort, individuals in the same type have the same ability to transform resources into outcomes. In his book, the focus is on equality of opportunity in education: educational outcome, student circumstances and effort are used to exemplify the theory

I propose that the equal-opportunity policy must equalize, in some average sense yet to be defined, the educational achievement of all types, but not equalize achievements within type, which differ according to effort.

Roemer's definition of equal opportunity can be formalised in a simple model. In a population of N individuals the individual outcome of interest y_i is the result of two sets of traits: circumstances beyond her control and responsibility characteristics. Circumstances belong to a finite set $\{c_1, \dots, c_k, \dots, c_n\} = \mathbf{C}$, and \mathbf{E} is the set of responsibility variables. Each circumstance c_k can assume ϕ_k values. All the possible combinations of values taken one at time from \mathbf{C} define a partition of the population into types. The partition is then made of a n types, where $n = \phi_1 \times \phi_2 \times \dots \times \phi_h$.

To simplify our notation we can use c to indicate the categorical variable which indicates which of the n types a given individual belongs to. Moreover for the moment we will assume that effort can be measured with a single scalar e .

For example if the two circumstances are sex (Male, Female) and race (White, Black) then c assumes $n = 2 \times 2 = 4$ values: $\{(White, Male), (Black, Male), (White, Female), (Black, Female)\}$.

This circumstance variable together with the effort exerted determine y , the outcome of interest:

$$y_i = g(c_i, e_i)$$

The combination of circumstances beyond individual control defines a partition of the population in n types: set of individuals sharing the same value of all circumstances, $Y = (y^{1,\cdot}, \dots, y^{k,\cdot}, \dots, y^{n,\cdot})$. Similarly, the effort exerted divides the population in m mutually exclusive groups defined as *tranches*, $Y = (y^{\cdot,1}, \dots, y^{\cdot,j}, \dots, y^{\cdot,m})$. Individuals sharing same circumstances and effort belong to the *cell* $Y = (y^{1,1}, \dots, y^{k,j}, \dots, y^{n,m})$.

Table 1 represents the distribution of outcome Y in our simplified example with two circumstances assuming two values each, three possible values of effort and twelve individuals. Each row of the matrix represents a type and each column a tranche.

Table 1: Y

c_1	c_2	c	$e = L$	$e = M$	$e = H$
W	M	1	$y_{1,1}$	$y_{1,2}$	$y_{1,3}$
W	F	2	$y_{2,1}$	$y_{2,2}$	$y_{2,3}$
B	M	3	$y_{3,1}$	$y_{3,2}$	$y_{3,3}$
B	F	4	$y_{4,1}$	$y_{4,2}$	$y_{4,3}$

The literature that has derived measure of inequality of opportunity from Roemer's theory has proposed to quantify inequality of opportunity as inequality within tranches. By construction these

measures assume value zero when all individuals exerting the same effort - in the same tranche - obtain the same outcome.

This definition of equal opportunity has been labeled ‘Roemer’s strong definition of equal opportunity’ (Lefranc et al., 2008; Ferreira and Gignoux, 2011). This definition has been translated in more than one measure of inequality of opportunity. One of the most well known and widely adopted is the *ex-post* measure of inequality of opportunity introduced by Checchi and Peragine (2010). They quantify unequal opportunity by measuring inequality in a counterfactual distribution obtained removing from the original distribution inequality between tranches. To this end they evaluate inequality in the counterfactual distribution \tilde{Y}_{EP} obtained replacing individual outcome with:

$$\tilde{y}_i^{k,j} = y_i^{k,j} \frac{\mu}{\mu^j}, \quad \forall i = 1, \dots, N \quad \forall k = 1, \dots, n \quad \forall j = 1, \dots, n.$$

Where I is a generic measure of inequality, $y_i^{k,j}$ is the outcome of individual i belonging to type k and exerting effort j , μ^j is the average outcome of tranche j , and μ is the average outcome of the population. Note that all tranches of the counterfactual distribution have the same mean, that is inequality between tranches has been removed. Inequality of opportunity is inequality in the counterfactual distribution:

$$IOp_{EP} = I(\tilde{Y}_{EP})$$

This measure is obtained following the three steps:

1. define the properties that an outcome distribution must have to satisfy the principle of equality of opportunity;
2. obtain a counterfactual distribution of which reflects the violations of those properties in the actual distribution;
3. measure inequality in the counterfactual distribution of unfair inequality.

All measures of inequality of opportunity that we will discuss shortly are obtained following the same three steps.

Note also that IOp_{EP} is a measure of inequality of opportunity based on relative outcome differences. If one is interested in an absolute approach, because he or she is dealing with health

inequality, for example, the counterfactual distribution can easily be expressed in absolute terms⁶. However, in the following we will always assume that we are interested in measuring inequality in relative terms.

The Roemer's strong definition of equal opportunity is a very demanding condition it implies the distributions of outcome conditional on effort to be identical for all types. A second, less demanding, definition of equal opportunity has been drawn from Roemer's theory. The 'weak equality of opportunity' criterion allows some inequality within tranches but requires that that mean advantage levels should be the same across types (Ferreira and Gignoux, 2011)⁷.

The *ex-ante* measure of inequality of opportunity proposed by Checchi and Peragine (2010) is a measure based on this weaker definition. The approach interprets the type-specific outcome distribution as the opportunity set of individual belonging to each type. The (utilitarian) value of the opportunity set of each type is the mean outcome of the type. Therefore inequality of opportunity in this case is simply between-type inequality, the counterfactual distribution \tilde{Y}_{EA} is obtained replacing individual outcome with:

$$\tilde{y}_i^{k,j} = \mu^{k,\cdot}, \forall i = 1, \dots, N \quad \forall k = 1, \dots, n \quad \forall j = 1, \dots, m$$

Where $\mu^{k,\cdot}$ is the mean outcome of type k .

$$IOp_{EA} = I(\tilde{Y}_{EA})$$

Adopting the *ex-ante* approach greatly simplifies the measurement of inequality of opportunity which becomes equivalent to measure between-group inequality, nevertheless IOp_{EA} is by far the most popular measure of inequality of opportunity⁸.

2.2 Responsibility Sensitive Egalitarianism

A similar approach is proposed by Fleurbaey and Maniquet; their proposal originates from a number of contributions on fair allocation and distributive justice (Fleurbaey, 1995; Fleurbaey,

⁶The *ex post* counterfactual distribution will be obtained as:

$$\tilde{y}_i^{k,j} = y_i^{k,j} - \mu^{i,j} + \mu, \quad \forall i = 1, \dots, N \quad \forall k = 1, \dots, n \quad \forall j = 1, \dots, n.$$

⁷An early contribution is Van de Gaer, 1993.

⁸Brunori et al. (2013) is a meta analysis of *ex-ante* inequality of opportunity measures in 41 countries. Today the number would be probably above 60.

2008; Fleurbaey and Maniquet, 2012). In these contributions the authors developed a theory of “responsibility-sensitive egalitarianism” whose ambition is to generalise the egalitarian ideal allowing individuals to be held responsible, to some degree, for their achievements. Again, individual well-being is determined by circumstances beyond individual control and individual choices, generally identified with a “responsibility variable”. Two ethical principles define a fair distribution of outcomes: the compensation principle and the reward principle. The first one states that any inequality due to circumstances beyond individual control is unfair and should be eliminated. The second specifies how final well-being should relate to responsibility characteristics. The principle of reward is absent from the original formulation of equality of opportunity proposed by Roemer and its meaning is less intuitive than the principle of compensation. The principle of reward clarifies what is the fair relationship between choices and outcomes. To what extent does a worker who exerts twice the effort of a colleague deserve a higher salary? Should she obtain twice as much as her colleague? Should the difference be determined by the difference in productivity?

There are many possible variants of the principle of reward. However, the majority of the authors implicitly or explicitly supplement the principle of responsibility with a principle of neutrality which prevents any redistribution that would go beyond the compensation of differences due to circumstances outside individual control. That is, given that inequality due to circumstances is zero, the fair outcome distribution is the distribution that naturally arises from effort exerted. If the principle of neutrality is agreed, then the definition of equal opportunity will satisfy the principle of compensation and the principle of *liberal reward* (Fleurbaey, 2008).

How can one measure inequality of opportunity within the responsibility-sensitive egalitarianism framework? The simplest discussion of this issue can be found in a paper published by Marc Fleurbaey and Eric Schokkaert (2009). Although this paper is about inequality in health, the same measures can be implemented for any dimension of well being. I suggest it as the first reference to understand the approach; a more complete discussion is developed in Fleurbaey (2008).

In order to make an inequality measure sensitive to the problem of responsibility, Fleurbaey and Shokkaert introduce two conditions:

Condition 1 (Reward, no influence of legitimate differences). A measure of unfair inequality should not reflect legitimate variation in outcomes, i.e. inequalities which are caused by differences in the responsibility variables.

Condition 2 (Compensation). If a measure of unfair inequality is zero, there should be no illegitimate differences left, i.e. two individuals with the same value for the responsibility variable should have the same outcome.

Putting together these requirements, we can state that a counterfactual distribution consistent with the compensation and the reward principles is a distribution that:

- 1) fully reflects the outcome inequality between individuals with the same effort (within-tranche inequality);
- 2) does not contain any outcome inequality between individuals characterized by same circumstances (within-type inequality).

Any inequality measure applied to such distribution would be a measure of opportunity inequality consistent with both the reward and the compensation principle.

Note that Condition 1 and Condition 2 are equivalent to impose precise properties to the counterfactual distribution. Condition 1 imposes that all individuals belonging to a type must be represented by the same value in \tilde{Y} .

$$\text{Property 1: } \tilde{y}_i^{k,l} = \tilde{y}_i^{k,h} = \lambda_k \quad \forall l, h = 1, \dots, m \text{ and } \forall k = 1, \dots, n, \text{ where } \lambda_k \in \mathbb{R}$$

This is a logical consequence of removing all inequality due to effort from the distribution: if we preserve some inequality within type in the distribution, we are including in inequality of opportunity some inequality which is due to choice.

Condition 2 imposes that the relative difference of values in the counterfactual distribution of all individuals belonging to the same tranche should be equal to the relative difference original outcome. That is to say we can write each tranche of the counterfactual distribution as the original distribution divided by a real number.

The intuition of this property is that relative inequality within tranches should be entirely reflected in the counterfactual distribution. Obviously this property could be translated in absolute terms if we were interested in absolute inequalities.

$$\text{Property 2: } s: \tilde{y}_i^{k,j} = \frac{y_i^{k,j}}{\gamma_j} \quad \forall j = 1, \dots, m, \text{ where } \gamma_j \in \mathbb{R} .$$

It can be easily shown that it is impossible to construct a counterfactual distribution that satisfies both conditions if the effect of circumstances on outcome is not independent of effort (See Fleurbaey (2008) for a discussion on the meaning of this incompatibility). More precisely, such a

counterfactual distribution can be obtained only if the function $g(c, e)$ is product separable in effort and circumstances⁹. A numerical example clarifies the point.

Table 2: Y

c	$e = L$	$e = M$	$e = H$
WM	3	4	4
WF	4	4	12
BM	4	6	11
BF	4	8	22

To satisfy Property 1 and Property 2 it should be possible to write the counterfactual distribution so that:

Table 3: \tilde{Y}

c	$e = L$	$e = M$	$e = H$
WM	$3/\gamma_1 = \lambda_1$	$4/\gamma_2 = \lambda_1$	$4/\gamma_3 = \lambda_1$
WF	$4/\gamma_1 = \lambda_2$	$4/\gamma_2 = \lambda_2$	$12/\gamma_3 = \lambda_2$
BM	$4/\gamma_1 = \lambda_3$	$6/\gamma_2 = \lambda_3$	$11/\gamma_3 = \lambda_3$
BF	$4/\gamma_1 = \lambda_4$	$8/\gamma_2 = \lambda_4$	$22/\gamma_3 = \lambda_4$

Where γ_j and λ_i are any real number.

Consider the first two cells of the first column: $3/\gamma_1 = \lambda_1$ and $4/\gamma_1 = \lambda_2$ then we know that: $\lambda_1/\lambda_2 = \frac{3}{4}$. However this is incompatible with what we obtain if we apply the same reasoning to the first two cells of the second column from which $\lambda_1/\lambda_2 = 1$.

This is an important point: if what individuals obtain from their effort depends on their circumstances it is impossible to find a measure of inequality of opportunity that both satisfies the compensation and the liberal reward principle.

IOp_{EP} for example is consistent with the compensation principle (Condition 2) but not with the reward principle (Condition 1). In fact the counterfactual distribution is obtained dividing the outcome in each tranche by $\gamma_j = \frac{\mu^j}{\mu}$. Unless the relative outcome of different types in all tranches is the same, some inequality within types will be reflected in \tilde{Y}_{EP} .

⁹Or is additively separable when we are dealing with absolute inequality.

Likewise, \tilde{Y}_{EA} satisfies Condition 1 ($\lambda_k = \mu^{k,\cdot}$). The counterfactual distribution does not reflect any inequality within types, but conflicts with Condition 2 because, unless the relative outcome of types is the same in all tranches, part of the inequality within tranches is not reflected in the counterfactual distribution.

The two measures proposed by Fleurbaey and Schokkaert (2009) explicitly attempt to find a solution to this issue by mitigating the conflict between the principle of reward and the principle of compensation. Each measure is fully consistent with one of the two and preserves consistency with the other for a reference degree of effort or a reference type.

Direct unfairness (DU): choose a reference value for the the responsibility variable \tilde{e} . Then $\tilde{y}_i^{k,j} = g(c_k, \tilde{e})$ is the level of outcome attained by individuals in type k if they exert the reference degree of effort. Inequality in the distribution of those outcomes, \tilde{Y}_{DU} , is inequality of opportunity.

Fairness gap (FG): choose a reference type \tilde{c} . Then $\tilde{y}_i^{k,j} = y_i^{k,j} / g(\tilde{c}, e_j)$, that is, inequality of opportunity is inequality in the distribution of initial outcome divided by the outcome of individuals in the same tranche with reference circumstance.

Note that IOp_{DU} measures inequality in a counterfactual distribution obtained by removing any inequality due to effort. All individuals belonging to the same type have the same value in \tilde{Y}_{DU} . Hence IOp_{DU} is a measure of inequality of opportunity fully consistent with the principle of reward (no influence of legitimate differences). On the other hand, IOp_{DU} is consistent with the principle of compensation for the reference degree of effort: if all individuals with the reference level of effort obtain the same outcome inequality in \tilde{Y}_{DU} is zero.

Symmetrically, IOp_{FG} measures inequality in a counterfactual distribution obtained by isolating inequality within tranches. It is a measure fully consistent with the principle of compensation: inequality in \tilde{Y}_{FG} is zero only if all individuals in the same tranche obtain the same outcome, irrespectively of their circumstances. Moreover, IOp_{FG} is consistent with the principle of reward for the reference circumstance; IOp_{FG} is insensitive to changes in inequality within individuals characterized by reference circumstances.

In the example in Table 2 IOp_{FG} and IOp_{DU} differ and more importantly, we can find transfers between individuals that imply changes in the opposite direction of the two indexes. Similarly, a conflict can arise between IOp_{EP} and IOp_{EA} .

Which counterfactual should be used? If we want our measure to always be consistent with the principle of reward, then the choice is between IOp_{EA} and IOp_{DU} . The former is attractive because we are used to understanding inequality as distance from the mean. IOp_{EA} looks at inequality between types' mean and is therefore more intuitive than IOp_{DU} which refers to a reference degree of effort. However, Direct Unfairness is to be preferred when we have in mind a particular level of effort to be the appropriate one. For example, when the outcome is health and the variable of responsibility is the diet, authorities may have in mind an appropriate caloric intake. In this case, IOp_{DU} is attractive because we can calculate the measure, setting the appropriate caloric intake as the reference responsibility variable. IOp_{DU} satisfies both the principle of reward and the principle of compensation for individuals choosing the recommended diet: when $IOp_{DU} = 0$ inequality between individuals following dietary guidelines is zero.

Similarly, when we want our measure to satisfy the principle of compensation, the choice is between IOp_{EP} and IOp_{FG} . Both measure inequality of opportunity as inequality within-tranche. However, the former is more intuitive because it defines within-tranche inequality in terms of distances from the mean, while the latter looks at distances from a reference combination of circumstances. That said, Fairness Gap may be more appropriate when we have a reference circumstance in mind because it also satisfies the principle of reward for individuals in that reference type.

In my opinion, one should preferably have good reasons to opt for Direct Unfairness and Fairness Gap. Although their axiomatic derivation is elegant, the resulting counterfactuals tend to be very sensitive to the reference effort/circumstances chosen. Consider again our simplified example:

Table 4: Y

c	$e = L$	$e = M$	$e = H$
WM	3	4	4
WF	4	4	12
BM	4	6	11
BF	4	8	22

Tables 5 and 7 show how inequality is reflected in the the ex-ante and direct unfairness counterfactual (when the reference effort is 3):

Table 5: \tilde{Y}_{EA}

c	$e = L$	$e = M$	$e = H$
WM	3.67	3.67	3.67
WF	6.67	6.67	6.67
BM	7	7	7
BF	11.33	11.33	11.33

Table 6: $\tilde{Y}_{DU}, \tilde{e} = 3$

c	$e = L$	$e = M$	$e = H$
WM	4	4	4
WF	12	12	12
BM	11	11	11
BF	22	22	22

To see how \tilde{Y}_{DU} can be sensitive to changes in the reference effort, consider what happens if the reference effort is the lowest:

Table 7: $\tilde{Y}_{DU}, \tilde{e} = 1$

c	$e = L$	$e = M$	$e = H$
WM	3	3	3
WF	4	4	4
BM	4	4	4
BF	4	4	4

Because when using \tilde{Y}_{DU} inequality of opportunity is inequality within the reference tranche large differences in the outcome distribution across tranches imply high sensitivity of the measure to the choice of the reference effort.

Tables 8 and 10 show the ex-post and fairness gap counterfactuals (when the reference combination of circumstances is WM).

Table 8: \tilde{Y}_{EP}

c	$e = L$	$e = M$	$e = H$
WM	0.67	0.73	0.33
WF	1.07	0.73	0.98
BM	1.07	1.09	0.90
BF	1.07	1.45	1.80

Table 9: $\tilde{Y}_{FG}, \tilde{c} = 1$

c	$e = L$	$e = M$	$e = H$
WM	1	1	1
WF	1.33	1	3
BM	1.33	1.5	2.75
BF	1.33	2	5.50

Consider what happens if the reference combination of circumstances is instead BF :

Table 10: $\tilde{Y}_{FG}, \tilde{c} = 4$

c	$e = L$	$e = M$	$e = H$
WM	0.75	0.5	0.19
WF	1	0.5	0.54
BM	1	0.75	0.5
BF	1	1	1

Within-tranche inequality is not affected, instead within-type inequality (or between-tranche) is much lower when $\tilde{c} = BF$. This comes from the fact that within-type inequality is higher in type 4. When choosing the reference circumstances we are also implicitly setting the relative difference in outcome due to effort that should not be considered inequality of opportunity.

To be consistent with the principle of compensation we will always violate liberal reward, that is to say we will always include some between-tranche (within-type) inequality as part of inequality of opportunity. In choosing the reference type, we impose which between-tranche inequality is not to be considered inequality of opportunity, but fair return to effort. The lower between-tranche inequality in the reference type, the higher the inequality in the \tilde{Y}_{FG} .

Consider two degree of effort $j < m$ and the difference $\tilde{y}^{k,j}/\tilde{y}^{k,m}$ reflected in \tilde{Y}_{FG} :

$$\tilde{y}^{k,j}/\tilde{y}^{k,m} = \frac{y^{k,j}}{y^{\tilde{c},j}} = \frac{y^{k,j}}{y^{k,m}} \frac{y^{\tilde{c},m}}{y^{\tilde{c},j}} = \frac{y^{k,j}}{y^{k,m}} \frac{1}{\frac{y^{\tilde{c},j}}{y^{\tilde{c},m}}}$$

Now look at how $\frac{\tilde{y}^{k,j}}{\tilde{y}^{k,m}}$ changes when $1/\frac{y^{\tilde{c},m}}{y^{\tilde{c},j}}$ varies:

- If $1/\frac{y^{\tilde{c},j}}{y^{\tilde{c},m}} = 1$ then the entire difference $y^{k,j}/y^{k,m}$ is reproduced in \tilde{Y}_{FG}
- If $1/\frac{y^{\tilde{c},j}}{y^{\tilde{c},m}} > 1$ only a share of the difference in the initial distribution will be considered
- if $1/\frac{y^{\tilde{c},j}}{y^{\tilde{c},m}} < 1$ more than the original inequality is reproduced in \tilde{Y}_{FG}

The last situation is especially counterintuitive: it shows that inequality in the counterfactual distribution can be higher than inequality itself. However, this situation is impossible in practice if we assume that the outcome is weakly increasing in effort $y^{k,j} \leq y^{k,m} \forall j \leq m$.

Finally, recall that the discussion above is based on an example in which we observe a single individual in each cell. Table 11 shows a more realistic situation in which each cell contains a different number of individuals. If we observe individuals sharing the same circumstances and exerting the same effort, how then should we consider such variability? Is inequality within-cell inequality due to effort or to opportunity?

Table 11: Y

c	$e = L$	$e = M$	$e = H$
WM	2, 3, 4	4	3, 5
WF	4	3, 5, 6	12
BM	4	6	9, 11, 13
BF	2, 6	8	22

Is it more likely that this inequality arises from unobservable effort or from unobservable circumstances? Is it simply a measurement error which is convenient to ignore, that is replacing all outcomes in the cell with their mean? The answer depends on our beliefs about circumstances and effort observability: Lefranc et al. (2009) consider within-cell inequality to be due to luck, a source of inequality of opportunity. On the contrary, the majority of the empirical contributions consider

inequality within cell as due to effort. Checchi and Peragine (2010), for example, claim that this inequality is due to limited effort observability and therefore should be attributed to effort.

In what follows we will endorse this latter view, assigning to each individual in type k exerting effort j the average outcome of cell k, j . For us, within-cell inequality is not inequality of opportunity. However, we will discuss in a few footnotes how one should proceed to include within-cell variability as part of inequality of opportunity.

In conclusion, we will discuss how to estimate four measures of inequality of opportunity—two of which are fully consistent with the principle of reward (IOp_{EA} and IOp_{DU}) and two of which are fully consistent with the principle of compensation (IOp_{EP} and IOp_{FG}). To obtain these counterfactuals, the outcome of the individual i belonging to type k and tranche j is replaced with a value \tilde{y}_i as follows¹⁰:

$$IOp_{EA}: \tilde{y}_i^{k,j} = \mu^{k,\cdot}$$

$$IOp_{EP}: \tilde{y}_i^{k,j} = \frac{\mu^{k,j}}{\mu^{\cdot,j}}$$

$$IOp_{DU}: \tilde{y}_i^{k,j} = \mu^{k,\bar{e}}$$

$$IOp_{FG}: \tilde{y}_i^{k,j} = \frac{\mu^{k,j}}{\mu^{\bar{e},j}}$$

¹⁰When within-cell variability is considered a source of inequality of opportunity:

$$IOp_{EA}: \tilde{y}_i^{k,j} = \mu^{k,\cdot} \frac{y_i^{k,j}}{\mu^{k,j}}$$

$$IOp_{EP}: \tilde{y}_i^{k,j} = \frac{y_i^{k,j}}{\mu^{\cdot,j}}$$

$$IOp_{DU}: \tilde{y}_i^{k,j} = \mu^{k,\bar{e}} \frac{y_i^{k,j}}{\mu^{k,j}}$$

$$IOp_{FG}: \tilde{y}_i^{k,j} = \frac{y_i^{k,j}}{\mu^{\bar{e},j}}$$

3 Estimation

Estimating a measure of inequality of opportunity requires two preliminary steps: 1) to identify outcome, effort and circumstances beyond individual control, 2) to manipulate the original distribution obtaining the counterfactual distribution that reflects inequality of opportunity.

Estimating a measure of inequality of opportunity requires two preliminary steps: 1) to identify outcome, effort and circumstances beyond individual control, 2) to manipulate the original distribution, thereby obtaining the counterfactual distribution that reflects inequality of opportunity.

3.1 Outcome, effort, circumstances

Inequality of opportunity should in principle be measured by looking at multidimensional welfare measures. The majority of authors do agree that individuals should be held responsible for their preferences on outcomes. If we measure inequality of opportunity in a single domain, we may include part of inequality of opportunity some differences in outcome due to preferences (see Decanq, et al. (2014) for a discussion). However, to measure inequality of opportunity taking into consideration the multidimensionality of welfare is complex. Moreover, as suggested by Roemer (1998), the level of abstractness of such a measure could limit its policy relevance. These considerations explain why the majority of the empirical applications focus on a single dimension of welfare.

Outcome

The typical outcome considered by the empirical literature is income (or consumption in poorer countries), but measures of inequality of opportunity have been proposed for many other dimensions: educational achievements (Gamboa and Waltenberg, 2012; Luongo, 2015), health (Li Donni et al, 2014), credit market (Coco and Pignataro, 2013), and even international aid (Llavador and Roemer, 2001), In principle one can imagine evaluating inequality of opportunity for any dimension of individual well being in which both circumstances and effort play a role. However, we should always remember that when evaluating inequality of opportunity in a single dimension, we are implicitly assuming that the outcome of interest is equally desirable for all individuals. The majority of empirical studies have selected income, a largely accepted measure of individual well being. Similarly, health and education may be easily considered appropriate outcomes. We cannot extend this reasoning to any outcome, however. It would be very misleading, for example, to measure inequality of opportunity in consumption of cultural goods because it is difficult to claim that

it represents an outcome that is equally desirable for everyone.

It is also problematic to measure inequality of opportunity for an outcome that cannot realistically be considered a joint result of circumstances and choice, because it is clearly due only to circumstances. For example, some scholars measure “inequality of opportunity of 10 year olds’ access to safe water” (Barros et al., 2013). Of course, it is difficult to hold children responsible for not having access to safe water. We will return to this point later when discussing effort.

Note that the choice of the outcome does affect the population of interest. If for example we were to choose individual earned income, it makes little sense to consider individuals out of the labour market in the analysis. It might be preferable to consider instead only the working population. Similarly, an estimate of inequality of opportunity in educational achievements should exclude all individuals that are still pursuing education. The choice of the outcome generally implies a selection of the sample that will no longer represent the original population. It represents a subsample of that original population with certain characteristics, typically the working age population.

Circumstances

The choice of circumstances is a key aspect of any empirical analysis. In principle, we would like to include all possible variables beyond individual control that affect the outcome. More precisely, we would like to include all circumstances beyond individual control that are sources of unequal opportunity. These two statements are not equivalent. Consider for example the variable ‘age’, clearly a circumstance beyond individual control and an important determinant of welfare. That said, should we include age among the sources of unequal opportunity? Should we instead consider the meaning of equal opportunity in a lifetime perspective and exclude demographic factors from the sources of unequal opportunity?¹¹. The distinction between sources of inequality of opportunity and sources of other types of inequality is critical and should be carefully considered case by case.

Unfortunately, in the applied literature the choice is largely driven by data availability. This issue is not particularly troublesome for richer countries where we are often able to observe a large

¹¹In some cases, even circumstances that do have an effect on the lifetime outcome may not be considered sources of unequal opportunity: “We should not consider males disadvantaged with respect to females if, due to innate biological factors, their life expectancy is shorter.” Roemer and Trannoy (2015) p. 277.

number of circumstances. Consider for example the case of the estimates proposed by Björklund et al. (2012) for Sweden. The set of observable characteristics they have been able to include in their analysis is impressive; it includes detailed information about parental education, parental income, body mass index during adolescence, intelligence quotient during adolescence and family structure. In poorer countries the number of observable circumstances is often severely limited. In both cases, only a subset of the circumstances that affect outcomes are observed.

Partial observability of circumstances is a source of bias for our measures. The literature has discussed this issue only in a limited number of cases. We should first distinguish between IOp_{EA} and the other three measures of inequality of opportunity for which it is necessary to observe effort.

If we observe a subset of the circumstances beyond individual control, IOp_{EA} is downward biased. To see why, consider the example of the previous chapter:

Table 12: Y

c	$e = L$	$e = M$	$e = H$
WM	3	4	5
WF	4	4	12
BM	4	6	11
BF	7	8	22

Now consider that it was possible to observe only race. The four columns would collapse to two (with two individuals in each cell) and, because each one of the new columns is the weighted average of the previous two, inequality would decrease¹².

Table 13: Y

c	$e = L$	$e = M$	$e = H$
W	3.5	4	8.5
W	3.5	4	8.5
B	5.5	7	16.5
B	5.5	7	16.5

This explains why IOp_{EA} estimate has been and should always be presented as a “lower bound” measure of the real level of inequality of opportunity.

¹²Provided that we use a Lorenz consistent inequality index (for a proof see Luongo (2011)).

The only cases in which we could get an upward biased estimate for IOp_{EA} is the case in which we include among circumstances variables of choice. A typical example is geographical area. If we observe area of birth, this is clearly a circumstance beyond individual control. However, if we observe area of residence this may not be a circumstance: the decision to migrate may be considered a way to exert effort in order to obtain higher outcome. Area of residence is only one of many possible examples, depending on which variable we are considering, this discussion can get very subtle. For example: can we consider religion a circumstance? In theory we are all free to embrace or to abandon a faith. Should we consider, then, inequality between religious groups unproblematic? Perhaps not. The discussion of this issue is an important one, but recall that the answer always depends on the view of the “social planner” or the “society”. In empirical applications the possibility that estimates are upward biased is warded off by choosing only circumstances that are clearly exogenous (race, area of birth, parental characteristics).

The sign and magnitude of the bias due to imperfect information about circumstances on $IOp_{EP}, IOp_{DU}, IOp_{FG}$ is not so immediately visible. We first consider the case in which effort is perfectly observable. If the degree of effort individuals exert is known, inequality of opportunity will again be a lower bound estimate of the real one. Consider again the example above (only race is observable) and recall how the three measures are obtained: IOp_{DU} is measured as inequality in m replications of one of the three columns (indicated by the reference degree of effort), IOp_{EP} is inequality in the distribution obtained by dividing each cell of Table 13 by the mean outcome of its column, and IOp_{FG} is inequality in the distribution obtained by dividing each cell by the outcome obtained by individuals in the same column and in the reference type. In all cases, inequality within columns is lower than the inequality that would be measured in the original columns of Table 12. Therefore when circumstances are not perfectly observable but effort is perfectly observable, the estimate of inequality of opportunity is again a lower bound of the real no matter which measure we adopt.

When neither circumstances nor effort are perfectly observable, the direction of the bias is instead ambiguous. Luongo (2011) for example shows that when the number of effort tranches of the counterfactual distribution is a subset of the real circumstances, IOp_{EP} may be upward biased. In this case, we can no longer claim that we are estimating a lower bound of the real inequality of opportunity. Effort observability is a demanding condition that is often violated. In our example, partial observability of effort is equivalent to a situation in which one is unsure about the column individuals belong. Clearly, a swap of columns can bias our estimate in both directions.

Finally, if we include one or more variables of effort in the set of circumstances, we will also

get a biased estimate of inequality of opportunity, no matter what measure we use. To get an intuition of why it is the case consider that to confuse an effort variable for a circumstance has the same effect than rearranging the distribution Y subtracting a column and increasing the number of rows; total inequality remains the same but we cannot predict the sign of the change in the within-column inequality (this could happen if we consider, for example, having a university degree as a circumstance).

Table 14 summarises the discussion. Circumstances (**C**) and effort (**E**) can be observable or only partly observable, and estimates can be unbiased, biased (unknown sign of bias) or lower bound. To include the effort variable among circumstances is the worst situation (last column).

Table 14: Measures' bias

measure	C & E	C & partly E	partly C & E	partly C & partly E	E in C
IOp_{EA}	unbiased	unbiased	lower bound	lower bound	biased
IOp_{EP}	unbiased	biased	lower bound	biased	biased
IOp_{DU}	unbiased	biased	lower bound	biased	biased
IOp_{FG}	unbiased	biased	lower bound	biased	biased

We should be very careful when comparing estimates of inequality of opportunity in different countries. Circumstance partial observability tends to bias downward our estimate. Circumstance observability may be correlated with the quality of the data and the quality of the data with many aspects of a country's economic development. We could get a low estimate of inequality of opportunity in a very poor country because of the quality of the data.

Effort

Once we have identified the circumstances beyond individual control, we should identify effort. In fact, it may be not necessary to observe effort if we limit our analysis to the estimation of ex-ante inequality of opportunity. In fact (IOp_{EA}) is based on the predicted outcome based on observable circumstances. This choice is made by the large majority of authors.

Roemer, for one, has proposed a method for identifying effort when it is not observable. His method is based on two assumptions. First the outcome is assumed to be monotonically increasing in effort. That is $y_{k,1} < y_{k,2} < \dots < y_{k,m}$ for $k = 1, \dots, n$. Second, the degree of effort exerted is by definition a variable orthogonal to circumstances. In Roemer's view, if individuals belonging to different types face different incentives and constraints in exerting effort, this is to be considered a characteristic of the type and therefore included among circumstances beyond individual control.

A student with well educated parents may find it much easier to spend hours sitting at her desk, while a student growing up in a less favourable environment may find it harder to study. Roemer believes that that the distribution of effort is, indeed, a characteristic of the type:

Thus, in comparing efforts of individuals in different types, we should somehow adjust for the fact that those efforts are drawn from distributions which are different, a difference for which individuals should not be held responsible.”

Roemer (2002) p. 458

Roemer therefore distinguishes between the ‘level of effort’ and the ‘degree of effort’ exerted by an individual. The latter is the morally relevant variable of effort and is identified with the quantile of the effort distribution for the type to which the individual belongs. In the example of effort exerted by students, the relevant measure is not the hours a student works but rather the quantile of type specific distribution of studying hours.

If effort is not observable but outcome is monotonically increasing in e we can identify the degree of effort exerted by a given individual with the quantile of the outcome distribution she sits at. The trick removes differences in effort that in Roemer’s view are due to circumstances beyond individual control and it makes it possible to compare the effort exerted by individuals in different types.

Note that this solution is impassable if the outcome of interest is measured with a dummy variable: access to a service, probability to survive, etc. In such cases we cannot construct the type specific distribution of outcome and therefore we cannot identify tranches. It may also be impossible in practice when the outcome is measured with a discrete variable which assumes few values.

In some cases effort variables may be at least in part observable. When measuring inequality of opportunity in health, for example, a number of behaviours may be considered a proxy for effort (smoking, drinking, physical activity). In such cases we can consider the effect of one or more of these variables on the outcome as due to effort.

Even if an effort variable is observable, such as smoking behaviour, one can decide to stick to the Roemer assumption of orthogonality of types and effort and again define the quantile of the smoking behaviour as the appropriate effort variable. The same identification strategy cannot be adopted when we observe more than one effort variable. If we observe both drinking and smoking

behaviour, for example, it is far from obvious how we should construct quantiles of the type specific distribution of such a two-dimensional effort.

When assuming that a certain variable is a responsibility characteristic, we should always bear in mind the meaning of our assumption. Consider one of the components of the Opportunity Human Development Index calculated by the World Bank: access to safe water for 10-year-old children. I personally consider it a bizarre exercise to estimate inequality of opportunity in such a dimension. Can we consider some part of the total inequality in access to safe water as due to responsibility variables? Can a 10-year-old be considered responsible for any outcome?

One of the questions here is at what age can we start to consider a child responsible for the effort she exerts. A good benchmark for such a discussion is the legal literature on criminal responsibility. After all, if at a certain age individuals are held legally responsible for their actions, the same principle could be implemented when measuring inequality of opportunity. In OECD countries the minimum age to be brought to court varies around an average of 13 years¹³. We should therefore be very careful when deciding to measure inequality of opportunity for children (many papers have for example exploited PISA data on 15-year-old students to assess inequality of opportunity in education). When a sufficiently rich dataset is available, one can try to overcome this problem, for example by including among circumstances the individual outcome before the age of responsibility. This is the choice of Brunori et al., (2013) in estimating inequality of opportunity in access to university. In the data used, there is information about school performance at different stages of the school curriculum. In that case, the test score at 19 years of age is considered the relevant variable of effort, but the degree of effort is defined as the quantile of the type specific test score distribution. And the test score at 14 years of age is included among circumstances. In doing so, one can partially control for the effort exerted when the student could not be considered responsible for her choice.

Measures of unequal opportunity for outcome of children have been justified on two distinct grounds. Firstly, if a child cannot be considered responsible for her choices, her parents may be. Secondly, it has been suggested that such measures are not intended to measure inequality of opportunity, but should rather be interpreted as measures of inequality in which inequality is more heavily weighted when correlated with circumstance beyond individual control. Suppose we have two societies with same average access to safe water for 10-year-olds; in one society the average access is the same for all ethnic groups, while in the other all individuals in ethnic minorities have no access to safe water. We should rank inequality in the latter more harmful than inequality in the

¹³More on this can be found in Melchiorre (2004).

former¹⁴.

The first point is reasonable, however, if the individual exerting effort is the parent and the outcome is her child's access to safe water. In that case, the analysis should be based on the parents' circumstances beyond individual control and not on the child's (I do not know any empirical application that chooses this solution). The second argument - that inequality correlated with circumstances should be considered more harmful - is also reasonable, but we should be aware that the estimate we obtain is not a measure of inequality of opportunity, but rather a measure of the degree of inequality in that coverage across type.

After outcome, circumstances and effort have been identified, the following step consists in estimating the counterfactual distribution. The counterfactual distribution, \hat{Y} , is the distribution of unfair inequalities. \hat{Y} can be estimated parametrically or non-parametrically. More rarely, it can be semi-parametrically estimated, an approach which is not covered here (for those interested, see Pistolesi (2009)).

After outcome, circumstances and effort have been identified the following step consists in estimating the counterfactual distribution. The counterfactual distribution, \hat{Y} , is the distribution of unfair inequalities. \hat{Y} can be estimated parametrically or non-parametrically. More rare is the case of semi-parametric estimation which is not covered here (if interested see Pistolesi (2009)).

3.2 Non-parametric approach

The non parametric approach, proposed first by Checchi and Peragine (2010) to estimate IOp_{EA} and IOp_{EP} , consists in partitioning the sample into types based on all observable circumstances, and then further partitioning each type into tranches (quantile of the type specific outcome distribution). In order to obtain the two counterfactual distributions (\tilde{Y}_{EA} , \tilde{Y}_{EP}) we must first estimate with precision the average outcome of each cell ($\mu^{k,j}$). This implies the need for a sufficient number of observations in each type, so that we can divide them into tranches (and for each tranche we would also like to have a sufficient sample size to estimate the average outcome). This can be difficult in practice. Recall that the number of types that partition the population is the combination of the values of all circumstances. If we have four circumstances assuming five values, we will have 625 types. With six circumstances assuming five values, we get 15,625 types, and the sample size of our data is likely to be smaller than this. The number of cells of a partition based on n dummies that describe circumstance and m tranches is $2^n \times m$.

¹⁴While many would agree on this point, a much more complicated issue is how average coverage and inequality between group can be traded-off in an aggregated measure.

Data availability dramatically drives the estimation of the counterfactual distribution when adopting a non-parametric approach. If we have a small dataset, an option is to estimate only IOp_{EA} . In this case we do not need to identify effort because inequality of opportunity collapses to a between-type inequality. For a reliable estimate we simply need a sufficient sample size in each type to calculate its mean outcome.

If instead we are interested in evaluating both reward-consistent and compensation-consistent measures of unequal opportunity, the only option we have is to limit the set of circumstances and tranches used. With a sample size of a few thousand, it is unrealistic to imagine a partition in more than some dozens of types. This is especially true because individuals are, as a rule, not uniformly distributed across the partition. If two or more circumstances are correlated it is very unlikely to observe some of their combinations. A typical case that arises when dealing with richer countries: we select parental education and parental occupation as circumstances. Unfortunately these two variables are strongly correlated and there are very few individuals whose parents are highly educated and employed in elementary occupations or who have no education but work as managers. Similarly, when dealing with poorer countries and including ethnicity and area of birth among circumstances: if ethnic groups are segregated in the country, we will find many empty types. Sparsely populated types are not a problem per se, but they threaten the reliability of the estimated counterfactual distribution.

It is clear therefore that in many cases we should limit the number of circumstances we use to define types. More frequently the values that describe circumstances are recoded reducing their variability. Districts of birth are aggregated in macro-region, parental occupations become a dummy for white/blue collar, ethnicity becomes a dummy for minorities. This point is particularly interesting because the literature has devoted a lot of attention to discuss the consequences of partial observability on our estimates and much less attention to the reliability of estimates. If one looks carefully at the empirical application, she may instead conclude that partial observability is a less relevant issue than estimate precision. Even when a comprehensive set of circumstances is available, we end up using only a subset of them.

What should guide our choice of circumstances, then? In principle we should give priority to the circumstance that explain the largest part of total inequality. A preliminary and quick exercise consists of running as many regressions as potential circumstances and looking at how R^2 changes after the exclusion of each variable. A proper way to obtain a reliable decomposition of total inequality by source in the context of inequality of opportunity measurement is discussed in Section 5.1.

3.2.1 Opportunity profile

When evaluating inequality of opportunity together with the typical tables of descriptive statistics of variables used in the study, there is another interesting figure worth including in the description of the data. The term *opportunity profile* introduced by Ferreira and Gignoux (2011) indicates a table containing information of types ranked by their mean level of advantage. This table generally includes for each type: the value of the circumstance variables, the average outcome, and the population share. Because in many empirical applications the mean outcome of small types can be calculated with low accuracy, I believe we should also include the sample size of each type and - with the risk of making the table less immediately readable - the standard error for the mean outcome. The aim of the opportunity profiles table is to give an intuition of what characteristics drive the ranking of types. And they can be extremely informative. Table 15 shows the important role of parental occupation in determining individual outcome in the United Kingdom.

Table 15: United Kingdom opportunity profile

rank	sex	parental occupation	parental education	sample	avg income	sd	population share
1	Male	elementary occupation	low	48	16,992.55	263.14	0.0069
2	Male	elementary occupation	high	4	17,787.69	2,234.30	0.0005
3	Female	Skilled manual	low	289	18,040.42	78.49	0.0426
4	Female	Highly skilled non-manual	low	1108	18,552.54	17.40	0.1576
5	Male	Skilled manual	high	87	18,575.54	160.14	0.0135
6	Female	elementary occupation	low	70	19,312.76	256.66	0.0096
7	Female	Lower skilled non-manual	low	508	19,738.67	33.33	0.0732
8	Male	Highly skilled non-manual	low	905	19,859.69	24.00	0.1445
9	Female	elementary occupation	high	7	19,965.84	1,987.68	0.0008
10	Male	Lower skilled non-manual	high	290	21,441.11	56.62	0.0443
11	Male	Skilled manual	low	231	21,829.35	133.14	0.0384
12	Female	Lower skilled non-manual	high	405	22,436.48	61.33	0.0609
13	Male	Lower skilled non-manual	low	408	22,559.41	56.63	0.0620
14	Female	Highly skilled non-manual	high	1158	24,609.11	23.44	0.1699
15	Female	Skilled manual	high	113	25,327.47	585.96	0.0159
16	Male	Highly skilled non-manual	high	967	25,847.95	34.45	0.1597

Source: *EUSILC, 2011*.

The opportunity profile is a natural way to present the data when estimating inequality of opportunity non-parametrically. In this case the profile is the counterfactual distribution \tilde{Y}_{EA} and has a direct link with the aggregated measure of unequal opportunity. Moreover, this way of presenting the data gives an idea of the reliability of the counterfactual distribution. A large number of

small types should make us suspicious about the reliability of analysis, especially when they tend to crowd into the bottom and top of the ranking (as type 2 in Table 15).

In Western countries the opportunity profiles typically show a large role of socioeconomic background: parental education and occupation heavily determines the ranking.

In poorer countries with a less usual socioeconomic structure, the role of parental occupation and education may be less important (and may be imprecisely measured). Other characteristics, such as birth location or ethnicity, can instead play a substantial role in shaping the opportunity profiles. This is the case of Ghana, for example, where individuals born in the North are disproportionately present among worst-off types.

Table 16: Ghana opportunity profile

<i>rank</i>	<i>ethnicity</i>	<i>birth location</i>	<i>parental education</i>	<i>sample</i>	<i>p.c. consumption 2013</i>
1	Kwa	north	none	793	917.40
2	others	north	none	282	923.73
3	Gur	north	none	11,519	1,103.89
4	Mande	north	none	244	1,285.88
5	Gur	centre	none	1,079	1,328.15
6	Gur	south	none	844	1,536.28
7	Gur	north	elementary or above	1,722	1,550.71
8	Mande	centre	none	78	1,561.71
9	Kwa	north	elementary or above	188	1,567.31
10	others	centre	none	147	1,570.04
11	Mande	south	none	67	1,683.66
12	others	centre	elementary or above	79	1,688.43
13	Mande	centre	elementary or above	36	1,731.18
14	Gur	centre	elementary or above	354	1,753.10
15	Kwa	south	none	7,852	1,792.07
16	Kwa	centre	none	2,962	1,907.49
17	Mande	north	elementary or above	23	1,980.65
18	Gur	south	elementary or above	363	2,154.66
19	Kwa	centre	elementary or above	3,715	2,181.44
20	others	north	elementary or above	34	2,257.19
21	others	south	none	185	2,330.63
22	Kwa	south	elementary or above	9,799	2,370.11
23	Mande	south	elementary or above	39	2,554.33
24	others	south	elementary or above	115	2,565.80

Source: Brunori et al., 2015.

In empirical contribution, the opportunity profiles are often used to identify the “worst-off” social groups. In particular Ferreira and Gignoux (2011) consider the ordered set of types, ranked by mean outcome, up until the type that brings the population share of the set over 10 percent. They call this subset of the opportunity profile the *opportunity-deprivation profile*. They show that in Latin America an important role is played by characteristics that identify ethnic or racial

minorities. Similarly, Brzezinski (2015) shows that first generation immigrants are largely overrepresented among the deprived types in Belgium.

Calculating the opportunity profile or the opportunity-deprivation profile is immediate. We simply calculate the average outcome and the population share.

3.2.2 Counterfactual distributions

The four counterfactual distributions to calculate inequality of opportunity are then obtained as follows:

- \tilde{Y}_{EA} is obtained replacing individual outcome with the type's mean outcome: $\tilde{y}_i^{k,j} = \mu^{k,\cdot}$.
- \tilde{Y}_{EP} is obtained dividing individual outcome by the tranche's mean outcome: $\tilde{y}_i^{k,j} = \frac{\mu^{k,j}}{\mu^{\cdot,j}}$.
- \tilde{Y}_{DU} is obtained selecting the reference tranche, and replacing individual outcome with the average outcome obtained by individuals in belonging to the same type and exerting reference effort: $\tilde{y}_i^{k,j} = \mu^{k,\bar{e}}$.
- \tilde{Y}_{FG} is obtained selecting the reference type and dividing individual outcome by the average outcome of individuals exerting the same degree of effort in the reference type: $\tilde{y}_i^{k,j} = \frac{\mu^{k,j}}{\mu^{\bar{e},j}}$.

3.3 Parametric approach

3.3.1 The basic model to estimate \tilde{Y}_{EA}

Burguignon et al. (2007) have proposed a regression-based method to estimate \tilde{Y}_{EA} . This method is the most adopted by the empirical literature, has been thoroughly discussed by Ferreira and Gignoux (2011) and is the method adopted by Wendelspiess and Soloaga (2014) for their Stata package IOP.

The approach estimates by OLS the outcome generating function:

$$y = g(\mathbf{C}, \mathbf{E}, u) \quad (1)$$

Where \mathbf{C} is the vector of circumstances, \mathbf{E} the vector of responsibility variables, and u is a random component which captures variation due to unobserved determinants. Because we know that effort is in part determined by circumstances the equation can be rewritten as:

$$y = g(\mathbf{C}, \mathbf{E}(\mathbf{C}, v), u) \quad (2)$$

Where v is again a random component.

As suggested by Burguignon et al. (2007) one can quantify the variability due to the vector of circumstances estimating by OLS the system of equations¹⁵:

$$y_i = \mathbf{C}_i\alpha + \mathbf{E}_i\beta + u_i \quad (3)$$

$$\mathbf{E}_i = H\mathbf{C}_i + v_i$$

Where α and β capture the direct effect of circumstances and effort on the outcome. H is a matrix of coefficients that capture the effect of circumstances on effort (and therefore their indirect effect on outcome).

Now, given that many circumstances are not likely to be observable, the error terms will not be orthogonal to regressors, and our coefficients' estimate will be biased. This is due to omitted variables and is another way to understand the problem of partial observability of circumstances discussed in Section 3.1. However, if we are not interested in the causal link between circumstances and outcome, but simply in identifying inequality of opportunity, we can use the reduced form of Equation 3:

$$y_i = \underbrace{\mathbf{C}_i\alpha}_{\text{direct effect}} + \underbrace{\mathbf{C}_i\beta\mathbf{H}}_{\text{indirect effect}} + \underbrace{v_i\beta}_{\text{effort effect}} + \underbrace{u_i}_{\text{residual}} \quad (4)$$

Which can be estimated by OLS as:

$$y_i = \mathbf{C}_i\Psi + \epsilon_i \quad (5)$$

Where $\Psi = \alpha + \beta\mathbf{H}$ and $\epsilon_i = v_i\beta + u_i$.

The ex-ante counterfactual distribution is simply the distribution of the predicted outcomes:

$$\tilde{Y}_{EA} = \mathbf{C}\hat{\Psi} \quad (6)$$

The explained variability of this regression model will capture both the direct effect of circum-

¹⁵In their original method, the functional form used is log-linear, probably due to its analogy to the Mincer equation, which has become a standard in this literature. However, as suggested by Ramos and van de Gaer (2015), the use of the log-linearized functional form is consistent with the majority of the axioms imposed to measure of inequality of opportunity only if we assume that the outcome of interest is the log of the variable considered. This could be the case for example of considering a logarithmic utility function.

stances and the indirect effect that circumstances play, through their effect on effort¹⁶.

Our estimates may still be biased if ϵ is correlated with C . This may not be a problem as long as omitted variables are unobservable circumstances. As discussed for the non-parametric approach, in this case what we obtain is a lower bound estimate of inequality of opportunity. Therefore, if the aim is to capture all inequality due to circumstances beyond individual control, biased coefficients for observable circumstances may improve rather than worsen our estimate.

Consider the example of inequality of opportunity in Rwanda, in which one can use the *Enquête Intégrale sur les Conditions de Vie des Ménages*, carried out by National Institute of Statistics of Rwanda for 2000. The survey contains information about birth location and socioeconomic background but does not include information about ethnicity, an important circumstance in Rwanda. When we estimate model (5) the effect of ethnicity will be in part captured by birth location and socioeconomic background if these variables are correlated with ethnicity. This implies that we should not interpret estimated coefficients. However, given that ethnicity is a circumstance, we should not be too concerned if we are simply interested in quantifying inequality due to circumstances. Although we do not observe ethnicity, at least some inequality due to ethnicity is captured by our measure of inequality of opportunity. That is, as long as omitted variables are circumstances, we will get a downward biased estimation of inequality of opportunity. Unfortunately, there are no reliable methods to quantify the magnitude of the bias, as Burguignon et al. (2007, 2013) note, and this aspect is often neglected by the empirical literature.

When estimating the ex-ante counterfactual, the only case in which we are unsure about the sign of the bias is when responsibility variables are correlated with circumstance. If, for example, individuals born in a given region tend to work harder, our estimate will capture some of the inequality due to hard work as inequality associated with birth location. Note however that this possibility can be ignored if we believe Roemer's assumption of orthogonality between circumstances and effort to be correct. According to Roemer, if individuals born in a given region work harder, this is a characteristic of their type and as such should be considered a source of inequality of opportunity and not a responsibility variable.

¹⁶Note that this would not be the case for a nonlinear model such as a Probit or Logit model (Roemer & Trannoy, 2015).

3.3.2 Effort identification

It is possible to obtain the other three counterfactual distributions (\tilde{Y}_{EP} , \tilde{Y}_{DU} , \tilde{Y}_{FG}) parametrically, but this would require us to identify effort. Effort can no longer be identified with the quantile of the type specific outcome distribution. In theory, types exist and are made of all individuals sharing the same value for all regressors, whereas in practice, especially when one or more circumstances are continuous, most of the types will be composed by a single individual.

We should distinguish between cases in which effort is observable and cases in which it is not. Moreover, effort can be unidimensional or multidimensional.

When effort is observable and unidimensional (3) it becomes:

$$y_i = \mathbf{C}_i\alpha + \beta e_i + u_i \quad (7)$$

$$e_i = \mathbf{C}_i\gamma + v_i$$

Individuals may be responsible for their effort (e_i) or for their degree of effort.

If individuals are held responsible for their absolute level of effort, we estimate the first equation and then we predict the counterfactual distributions as:

$$Y_{EA} : \tilde{y}_i^{k,j} = \mathbf{C}_i\hat{\alpha} \quad (8)$$

$$Y_{EP} : \tilde{y}_i^{k,j} = \frac{\mathbf{C}\hat{\alpha} + \hat{\beta}e_i}{\bar{\mathbf{C}}\hat{\alpha} + \hat{\beta}e_i} \quad (9)$$

Where $\bar{\mathbf{C}}$ is an average of circumstances across all individuals. Going back to our numerical example, imagine that instead of one individual per cell we have different population shares: black individuals make up 65% and women 55%. Equation (3.7) will be: $y_i = \alpha_1 black_i + \alpha_2 women_i + \beta e_i + u_i$. Then Y_{EP} is obtained dividing outcomes by $0.65\hat{\alpha}_1 + 0.55\hat{\alpha}_2 + \hat{\beta}e_i$.

Note that this counterfactual distribution is not exactly the one proposed by Checchi and Peragine (2010). We are not dividing the individual outcome by the average outcome of individual in the tranche. Instead we are dividing the individual outcome by the average outcome that the population would get if they were all exerting the same level of effort she exerts. If individuals characterized by different circumstances exert different levels of effort, then the two levels of effort differ.

Moreover, if instead of a linear specification we are using a nonlinear specification, we should define $\bar{\mathbf{C}}$ as the combination of circumstances that produces the average outcome, and it will no

longer be the combination of average circumstances. Think for example at the possibility to introduce in our regression model a circumstance squared to capture some nonlinear relationship. In this case, the value of the variable that produces the average outcome is no longer the average value in the distribution, but the root mean square of it.

The direct unfairness counterfactual is obtained by predicting the outcome of all individuals if they had exerted the reference effort \tilde{e} .

$$Y_{DU} : \tilde{y}_i^{k,j} = \mathbf{C}_i \hat{\alpha} + \hat{\beta} \tilde{e} \quad (10)$$

The fairness gap counterfactual is obtained by dividing predicted individual outcome by the predicted outcome of individuals exerting the same level of effort and with reference combination of circumstances $\tilde{\mathbf{C}}$.

$$Y_{FG} : \tilde{y}_i^{k,j} = \frac{\mathbf{C} \hat{\alpha} + \hat{\beta} e_i}{\tilde{\mathbf{C}} \hat{\alpha} + \hat{\beta} e_i} \quad (11)$$

Where $\tilde{\mathbf{C}}$ is the reference combination of circumstances which identifies the reference type.

If individuals are held responsible for their degree of effort we first estimate the second equation of (7).

$$e_i = \mathbf{C}_i \gamma + v_i$$

The estimated residual \hat{v}_i can be interpreted as the effort exerted after we have removed the effect of circumstances on effort, which Roemer calls ‘the degree of effort’. We then estimate:

$$y_i = \mathbf{C}_i \alpha + \hat{v}_i \gamma + u_i \quad (12)$$

And use predicted outcomes for different combination of circumstances and effort to predict the counterfactual distributions (8), (9), (10), (11). Note that this modifies the vector of coefficients α which now captures both the direct and the indirect effect of circumstances on outcome. If e and \mathbf{C} are correlated using the degree of effort \hat{v}_i instead of the level of effort e_i , it implies, *ceteris paribus*, that more variability will be explained by circumstances. This is consistent with the fact that now we are considering individuals only partly responsible for the consequences of their absolute level effort.

If more than one variable of effort is observable, we should follow different strategies. Li Donni

et al. (2014), for example, model a measure of health status as a function of both circumstances and effort variables. They consider three variables of effort: being a smoker, having high education, having high economic status. They obtain eight effort tranches, a combination of the three dummy variables. They then use the average outcome in each tranche to rescale individual outcome so as to obtain the ex post counterfactual distribution \hat{Y}_{EP} . Using the same strategy, one can also construct \hat{Y}_{DU} and \hat{Y}_{FG} , choosing a reference effort tranche for the former and selecting a reference combination of circumstances for the latter. This example clarifies that we do not need to have ordinal variables of effort in order to implement these measures. Categorical variables are ok as far as we are able to identify effort tranches.

In most of the empirical cases, variables of effort are not directly observable. If and when this is the case, we should focus on the residuals of the regression (5):

$$\hat{\epsilon}_i = y_i - \mathbf{C}_i \hat{\Psi} \quad (13)$$

ϵ captures a mix of measurement error, luck, and effort, but is generally interpreted as effort as long as it is orthogonal to circumstances. This is not generally the case: residuals are heteroskedastic, their variability tends to correlate with circumstances.

Here we find the same problem we encounter when following a non-parametric approach. We observe heterogeneity in the distribution of outcome across types and we know that a part of this heterogeneity is due to the indirect effect of circumstance on outcome (through effort). In the non-parametric approach, the problem is solved considering the quantile of the outcome distribution in order to identify a measure of degree of effort orthogonal to circumstances. In the parametric approach, it is generally impossible to estimate the outcome distribution of individuals sharing the same value of all regressors.

This solution is adopted by Biörklund et al. (2012). They predict the type-specific variance of the residual and use it to standardize the residual so as to remove heteroskedasticity. Only the standardized residual is then considered to be due to effort while the variability of the residual correlated to circumstances is considered to be due to inequality of opportunity. Biörklund et al. (2012), however, have a very large sample: they have the entire population of Swedish men born between 1955 and 1967. Furthermore, they manipulate the variables describing circumstance in order to get ‘few’ discrete variables which partition the sample into 1,152 types.

In the same vein one can identify the degree of effort by removing the part that is explained by regressors from the variability of the residual.

$$|\hat{\epsilon}_i| = \mathbf{C}_i\delta + \pi_i \quad (14)$$

Where δ is a vector of coefficients that capture the effect of circumstances on the absolute value of the residual ϵ . I am not aware of any empirical estimates based on this definition of effort.

3.4 Nonparametric Vs. parametric estimates

Parametric estimation has been proposed as a good alternative to nonparametric estimation when the sample size does not allow to reliably estimate types' mean outcome. If circumstances are correlated the problem can persist even when a large sample is available. The parametric approach is more parsimonious because it requires to estimate the average effect of a certain circumstance on the outcome. Moreover, this allows us to include more circumstances than we could include adopting the nonparametric approach. Region of birth could be replaced with district of birth without fear of not getting significant coefficient for the variables. Similarly years of education can substitute a dummy variable for a high/low level of parental education.

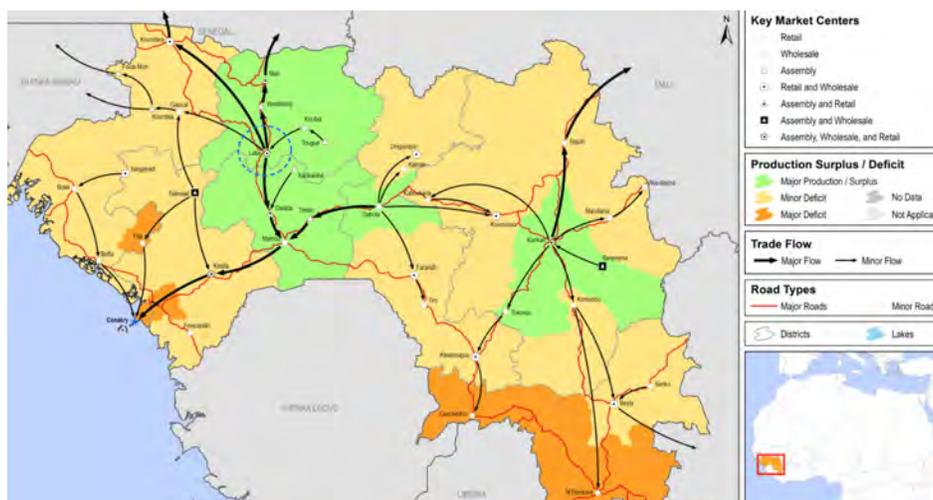
This parsimony is obtained at the cost of a less flexible structure imposed on the function $g(c, e)$ generally assumed linear or log linear (due to its similarity with the Mincer equation). Table 17 is a good example of the problem we may encounter following a parametric approach. In Guinea, among the observable circumstances in the *Enquête Intégrée de Base pour l'Évaluation de la Pauvreté* (2003) there is area of birth and father occupation (agriculture or other). In the table, information is collapsed in order to clarify how the two circumstances interact. In the rest of the country, having a father employed in agriculture is associated with lower consumption. By contrast, for individuals born in the region of Labe, being the child of someone working in agriculture means, on average, to have higher consumption later in life. Not surprisingly, the Labe region is an important centre of national and international agricultural trade flows.

Table 17: Non-linear impact of circumstances: the case of Guinea

birth place	parental occupation	per capita consumption
rest of Guinea	agriculture	843.10
rest of Guinea	other	1,117.60
Labe	other	1,272.88
Labe	agriculture	1,805.57

Source: Brunori et al., 2015.

Figure 1: Trade flows in Guinea (Fonio)



Source: FEWS (2013) p. 4

Such a problem can be easily solved by introducing an interaction term: $parental\ occupation \times birth\ place$ in the regression. However, if on the one hand we can include interaction terms and nonlinear transformation of original variables among the regressors, on the other hand we must be aware that this will reduce model parsimony and will make our parametric approach very similar to the non-parametric approach¹⁷. This is especially true if regressors are non-cardinal variables. Gender, ethnicity, area of birth, and parental occupation are categorical variables. To run a regression on non-cardinal variables, we have to transform them into sets of dummies. Each dummy will capture a shift in the intercept of the regression line which represents the advantage/disadvantage of being characterized by a certain value of the circumstance. In such a regression, if we were to interact all circumstances we would be estimating an intercept for all combination of circumstances—that is, we are back to non-parametric approach.

For these reasons, non-parametric estimates of inequality of opportunity are preferable to parametric estimates, which are themselves preferable when cardinal circumstances are observed. Parental income serves as a good example: in a non-parametric approach, we would need to partition parental income into quantiles in order to obtain types with a large sample size. In such a case, one might better approximate the effect of parental income on the offspring outcome by estimating an OLS in which parental income and some nonlinear transformation of it are included among

¹⁷See Hufe and Pichl (2015) on the effect assuming linearity when estimating parametrically inequality of opportunity.

regressors.

4 Inequality of opportunity index

The large majority of the contributions have proposed a synthetic index of equal opportunity. Such indexes are obtained applying an inequality measure to one of the counterfactual distributions. Following the methodology proposed by Checchi and Peragine (2010) the largest part of the authors use the mean logarithmic deviation (MLD).

$$MLD(y) = \frac{1}{N} \sum_{i=1}^N \ln \frac{\mu}{y_i} \quad (15)$$

Note that MLD is the generalised entropy index when its parameter is set to zero. The reason why it was proposed is that it is a path-independent perfectly decomposable inequality index. As shown by Foster and Shneyerov (2000) the only inequality measure that satisfies path independence decomposability, uses the arithmetic mean as the reference, and that satisfies the Pigou-Dalton transfer axiom is the MLD¹⁸.

This property is exploited in empirical application to decompose total inequality in two parts: the share due to opportunity and a residual. If we have a population partitioned into n subgroups (types):

$$MLD(y) = MLD(\mu_1, \dots, \mu_k, \dots, \mu_n) + \sum_{k=1}^n w_k MLD(y_i)_{i \in k} \quad (16)$$

Where w_k is the type k 's population share and $(\mu_1, \dots, \mu_k, \dots, \mu_n) = \tilde{Y}_{EA}$. It follows that total inequality can be decomposed into inequality of opportunity and a residual term. Moreover, it makes sense to calculate what is the share of inequality due to opportunity:

$$IOP_{EA}^R = \frac{MLD(\tilde{Y}_{EA})}{MLD(y)} \quad (17)$$

The same kind of decomposition can be obtained for the compensation consistent measure (ex-post). In this case we have:

¹⁸Path independence means that inequality between (within) groups can be calculated both directly or subtracting the within (between) groups inequality from total inequality. Both paths yield the same values.

$$MLD(y) = MLD(\mu^1, \dots, \mu^j, \dots, \mu^m) + \sum_{j=1}^m w_k MLD(y_i)_{i \in j} \quad (18)$$

Where $(\mu^1, \dots, \mu^j, \dots, \mu^m)$ is the distribution of tranches' mean and $\sum_{j=1}^m w_k MLD(y_i)_{i \in j} = \tilde{Y}_{EP}$, therefore:

$$IOp_{EP}^R = \frac{MLD(\tilde{Y}_{EP})}{MLD(y)} \quad (19)$$

Beside its perfect decomposability the MLD has two main limitations: it is not very intuitive and it is unbounded. Two weaknesses rarely acknowledge by the literature.

An other inequality index (less frequently) used is the Gini coefficient:

$$\text{Gini}(y) = \frac{1}{N} \left(N + 1 - 2 \frac{\sum_{i=1}^N (N + 1 - i) y_i}{\sum_{i=1}^N y_i} \right) \quad (20)$$

In this case we can measure inequality of opportunity as the Gini of the counterfactual distribution. The meaning of the number obtained, and its relationship with the Lorenz curve of the counterfactual distributions, is well known. However, whenever the ranges of the groups specific outcome distribution present some overlapping we know that Gini is not perfectly decomposable.

In the case of ex-ante inequality of opportunity for example we get:

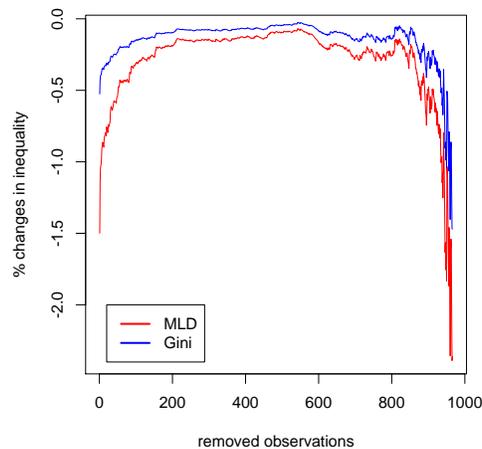
$$\text{Gini}(y) = \text{Gini}(\tilde{Y}_{EA}) + \sum_{k=1}^n a_k w_k \text{Gini}(y_i)_{i \in k} + K \quad (21)$$

Where a_k is the outcome share of type k and $K > 0$ whenever the group-specific distributions of outcome overlap. In this case we cannot identify the share of inequality due to opportunity because K is technically nor part of the between-group neither of the within-group inequality.

Recently Brunori et al. (2015) and Checchi et al. (2015) have noticed that estimates of the share of total inequality due to opportunity based on MLD tend to be much lower than estimates obtained with Gini. This is counterintuitive because given the non-decomposability of the Gini we would expect the Gini between types (and within tranches) to capture a smaller share of total inequality (the residual is not in the nominator but is part of the denominator). Brunori et al. (2015) suggest that this is due to the higher sensitivity of the MLD to outlier values. When constructing the counterfactual distributions we are by definition removing extreme values from total inequality.

This smoothing reduces more inequality as measured by an index more sensitive to extreme values. To get an intuition of this consider Figure 2 which was obtained simulating a lognormal distribution of 1,000 observations with mean 0 and standard deviation 1. At each step the highest observation is subtracted from the distribution and both MLD and Gini are calculated. What is plotted in the vertical axis of Figure 2 is the percentage change in each one of the two measures at each step (horizontal axis). Overlooking the last steps in which there are few observation and changes in inequality become random, what we can see is that the reduction in MLD is always higher in absolute term than the reduction in the Gini. This property - never proved by the literature to the best of my knowledge - does not depend on the standard deviation of the distribution and holds also for a normal distribution.

Figure 2: Sensitivity to extreme values: MLD Vs. Gini



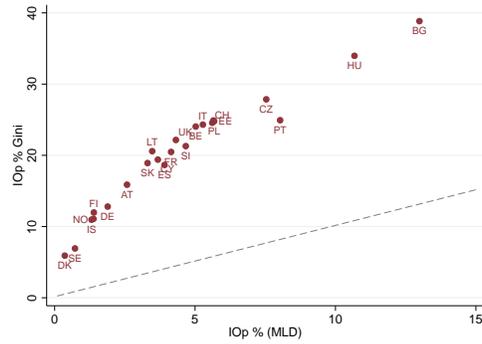
1,000 observations lognormally distributed with $\mu = 0, \sigma = 1$.
At each step the highest value is removed from the sample.

This property is shown in Figure 3 which reports share of total inequality due to opportunity in Europe calculated using both Gini and MLD. MLD between types captures less than half of the inequality captured by Gini between types.

The fact that Gini and MLD produce very different results is worrisome. Which inequality index should be preferred in empirical applications?

My personal opinion is that the Gini is to be preferred not only because its meaning is well known but also because MLD seems to be very sensitive to the removal of extreme values. Moreover, recall that we can always compare total inequality in the direct unfairness gap and direct

Figure 3: Share of total inequality due to opportunity: MLD Vs. Gini in Europe



Source: Brunori (2015), IOp_{EA} is based on EU-SILC data 2011.

unfairness distributions but it makes no sense to try to identify the share of inequality due to opportunity that could be higher than 100%.

5 Extensions

5.1 Inequality of opportunity decomposition by sources

Ferreira and Gignoux (2011) suggest that “[...] the parametric approach might permit the estimation of the partial effects of one (or a subset) of the circumstance variables, controlling for the others” p. 16. We follow exactly their example: IOP_{EA} is estimated parametrically with a log-normal OLS model and inequality is evaluated with MLD.

$$\ln y = \mathbf{C}\hat{\Psi} + \epsilon \quad (22)$$

$$\tilde{Y}_{EA} = \text{Exp} \left\{ \mathbf{C}_i \hat{\Psi} \right\} \quad (23)$$

Given the decomposability properties of MLD Inequality of opportunity can be measured following two paths:

$$IOP_{EA} = \text{MLD} \left(\mathbf{C}\hat{\Psi} \right) \quad (24)$$

or

$$IOP_{EA} = \text{MLD} (Y) - \text{MLD} \left(\tilde{V} \right) \quad (25)$$

Where $\tilde{V} = \bar{\mathbf{C}}\hat{\Psi} + \hat{\epsilon}$ and $\bar{\mathbf{C}}$ is an average of circumstances across all individuals.

The marginal contribution of each circumstance can then be obtained exploiting the last definition by constructing an alternative counterfactual distribution:

$$\tilde{V}_i^j = \text{Exp} \left\{ \bar{\mathbf{C}}_i^j \hat{\Psi}^j + \mathbf{C}_i^{k \neq j} \hat{\Psi}^{k \neq j} + \hat{\epsilon}_i \right\} \quad (26)$$

In this counterfactual distribution all circumstances takes their actual values but circumstance j which equalized to its average for all individuals.

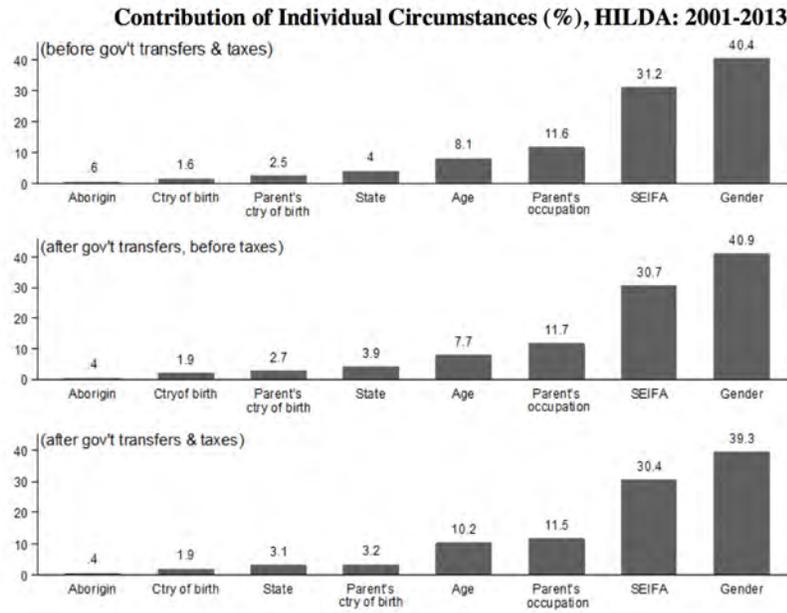
$$IOP_{EA}^j = \text{MLD} (Y) - \text{MLD} \left(\tilde{V}^j \right) \quad (27)$$

Is inequality of opportunity due to opportunity j .

For the case of parametric ex-ante inequality of opportunity based on MLD, partial contributions are easily calculated using the package **io** developed by Wendelspiess and Soloaga (2014). Figure 4 reports the decomposition of total inequality of opportunity by sources as estimated for Australia by Martinez et al. (2015) using the routine **io** for STATA (see the next paragraph for

details).

Figure 4: IOp_{EA} decomposed by source: Australia 2001-2013



Source: Martinez et al., 2015 p. 25.

5.1.1 iop STATA routine

The routine **iop** written for STATA by Wendelspiess & Soloaga (2014) can be used to estimate a number of inequality of opportunity measures.

The default command is:

iop *dependant_variable independent_variables*

It returns three values:

Where Ferreira-Gignoux without scale is the R^2 of the OLS regression (explained variance over total variance). This last measure is considered by Ferreira and Gignoux (2011b) as the most appropriate measure when dealing with a continuous variables that are the result of a standardization, test scores in education for example. This because in these cases a MLD and Gini of the pre and post standardization distributions are not ordinal equivalent, that is the ranking in terms of inequality of two distribution may differ before and after standardization. The variance instead is not problematic from this perspective and is also additive decomposable. However, using the variance the

Table 18: **iop** routing default output

	absolute	relative
Ferreira-Gignoux (with scale)	$IOP_{EA} = MLD(\mathbf{C}\hat{\Psi})$	$IOP_{EA}^R = \frac{MLD(\mathbf{C}\hat{\Psi})}{MLD(Y)}$
Ferreira-Gignoux (without scale)		$IOP_{EA}^R = \frac{VAR(\mathbf{C}\hat{\Psi})}{VAR(Y)}$

absolute measure is not interpretable because its value depends on the standardization method.

A number of options are allowed, among them:

detail: shows the output of the regression used to estimate the counterfactual distribution,

bootstrap(number of samples): calculates bootstrapped standard error of the inequality of opportunity measure,

shapley(independent_variables): decomposes the inequality of opportunity measure in the relative contribution of each circumstance variable based on the Shapley value (this, under very strong assumptions, is equivalent to identify the relative causal effect of each circumstance on outcome).

Using the option *shapley* one can obtain the decomposition proposed by Ferreira and Gignoux and discussed in the previous paragraph. Because the Shapley value decomposition is computationally very intensive it is advisable to group circumstances using the option *sgroups(circumstance1 circumstance2, circumstance3,...*). One can for example group all circumstances describing parental occupation or group variables describing regions in set of variables describing macro areas.

5.2 Human Opportunity Index

The World Bank has recently started to compile the Human Opportunity Index (HOI), a statistic based on the coverage rate of a number of essential services. Estimates of HOI were published for the first time in Paes de Barros et al. (2009). Here we follow Chapter 2 of Dabalen et al. (2015) which is a good introduction to the methodology, a complete discussion can be found in Paes de Barros et al. (2010).

The HOI is calculated for the access of children to basic services and goods which can be considered prerequisites needed for childhood development. The list may varies depending on data availability but generally include access to safe water, access to electricity, access to primary education, access to immunization.

The authors define HOI a measure of “equality of opportunity-sensitive coverage rate”. This means that HOI for a certain basic need increases with overall coverage and decreases with inequality in coverage due to circumstances beyond individual control.

To construct an index with these two properties they calculate the average coverage rate \bar{Y} and a measure of between-type inequality in access to the service, D . HOI is then:

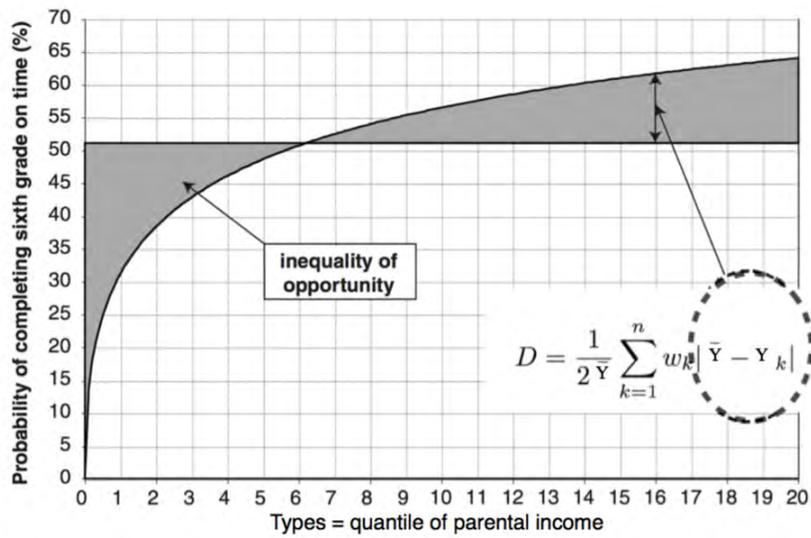
$$HOI = \bar{Y} \times (1 - D)$$

The between-type inequality measure is obtained first identifying circumstances beyond individual control and partitioning the population in types. Then the average access rate of each type, Y_k is used to calculate D :

$$D = \frac{1}{2\bar{Y}} \sum_{k=1}^n w_k |\bar{Y} - Y_k| \quad (28)$$

Where w_k is the share of group k in total population and n is the number of types. The index D is reward-consistent measure of inequality of opportunity, more precisely is an ex-ante measure of inequality of opportunity because it evaluates inequality of opportunity as inequality in the counterfactual distribution made of type-specific coverage rates. The dissimilarity index can be interpreted looking at Figure 5 which plots the percentage of individuals completing sixth grade education on time (vertical axis) as function of the ventile of the income distribution their household belongs to (horizontal axis). The horizontal line (slightly above 50 percent) represents the percentage in the entire population that completed sixth grade on time. The point where the type-specific coverage rates curve and the average coverage line cross divides the population into two groups: the vulnerable (or worse-off) types, with a coverage rate below \bar{Y} and the non-vulnerable (or better-off) types,

Figure 5: A graphical representation of the dissimilarity index D



Source: Adapted from Paes de Barros, 2009 p. 65.

with a coverage above the average. The shaded area represents the sum of gaps between type-specific percentage and the overall percentage. The left-hand shaded area can be interpreted as “the fraction of available opportunities that need to be reassigned from better-off groups to worse-off groups to achieve equal opportunity for all.” Paes de Barros (2009) p. 65. The right-hand shaded area is by definition equal to the left-hand shaded area (and this explains why the sum of the gaps is divided by two in equation 28).

HOI have a number of properties:

a) HOI ranges between \bar{Y}^2 and \bar{Y} . Takes value \bar{Y} when $D = 0$, that is when $Y_k = \bar{Y} \forall k = 1, \dots, n$. It takes value \bar{Y}^2 when the coverage is 100% for \bar{Y} share of the population and is 100% for $(1 - \bar{Y})$ share of the population.

b) HOI is sensitive to the coverage scale: if the coverage increases (decreases) by a factor δ , HOI increases by the same factor.

c) HOI is sensitive to redistribution: if the coverage of a non-vulnerable type is decreased in favour of a vulnerable type, holding constant the average coverage in the population, HOI does not

decreases¹⁹.

d) HOI is insensitive to redistribution among vulnerable or non-vulnerable types.

e) sub-group inconsistency: HOI cannot be decomposed into HOI calculated for subgroups of the population. Take the example of regions in a country, it can be the case that HOI declines in a region, remains the same in all the other regions, but increases for the entire population. The intuition of this is that the effect of transfers of coverage between types depends on whether they are worse-off or better-off types. A type may be better-off in a subgroup but worse-off in the population.

Property d) and e) may be considered undesirable but they can be solved calculating the Geometric Human Opportunity Index (G-HOI). G-HOI is obtained as the geometric mean of the type-specific coverage rates. It is both strictly sensitive to redistribution and subgroup-consistent.

An other interesting property of this index is that changes in HOI can be decomposed in: 1) a *scale effect* which refers to changes in the average coverage in the population, 2) a *distribution effect* which refers to changes in the between-group inequality in coverage holding constant the coverage rate in the population.

Consider the change of HOI over time:

$$\Delta HOI = HOI_{t1} - HOI_{t0} = \bar{Y}_{t1}(1 - D_{t1}) - \bar{Y}_{t0}(1 - D_{t0})$$

Adding and subtracting $\bar{Y}_{t1}(1 - D_{t0})$ we get a decomposition:

$$\text{scale effect : } \bar{Y}_{t1}(1 - D_{t0}) - \bar{Y}_{t0}(1 - D_{t0})$$

and

$$\text{distribtuion effect : } \bar{Y}_{t1}(1 - D_{t1}) - \bar{Y}_{t1}(1 - D_{t0})$$

Azevedo et al. (2010) have developed a STATA package to estimate and decompose HOI. The estimation of Y_k is based on a parametric estimation of the conditional probabilities of access the service given a vector of circumstances (C). A logistic model is estimated:

¹⁹And it increases if the non-vulnerable from which some coverage is subtracted does not become vulnerable.

$$\ln \left[\frac{\Pr \{y = 1|C\}}{1 - \Pr \{y = 0|C\}} \right] = C\Psi \quad (29)$$

The coverage of individual i with circumstances C_i can then be predicted using the estimated coefficients:

$$\hat{y}_i = \frac{\text{Exp} \{C_i\Psi\}}{1 + \text{Exp} \{C_i\Psi\}} \quad (30)$$

And used to calculate HOI:

$$HOI = \hat{Y} \left(1 - \frac{1}{2\hat{Y}} \sum_{n=1}^N w_n |\hat{Y} - \hat{y}_i| \right) \quad (31)$$

For a sufficiently low number of types we could estimate non-parametrically HOI simply calculating the average coverage for each type. The authors of the package however have chosen to adopt the parametric approach and suggest not to include interaction terms in the analysis especially if the sample size is small (Dabalen et al., 2015).

HOI can be calculated using the STATA routine **hoi**:

hoi dependent_variable independent_variables

It returns a table of results which include: the average coverage rate (\bar{Y}), the dissimilarity index (D), *HOI*, the share of vulnerable population and part of the output of the logit regression used to predict type-specific coverage rates.

The use of the option *, by* returns a counterfactual using the coefficients from an alternative period in time or place.

Finally, a composite HOI index has been proposed in order to obtain a single scalar which aggregates access to multiple goods and services for children. To compute the composite HOI one has to construct a variable which takes value 1 if the child is covered by all the goods and services and takes value 0 otherwise. Then the composite HOI is computed with the same formula (5.2).

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